CS-171, Intro to A.I., SS-1, 2018 — Quiz # 2 — 20 minutes

NAME:____

YOUR ID: __________ ID TO RIGHT:__________ ROW NO.:______ SEAT NO.:________

1. (40 pts total, -10 for each error, but not negative) RESOLUTION THEOREM PROVING.
(From http://www.iqtestexperts.com/brainteasers/index.php/2011/05/21/who-broke-the-pot/)
Three friends Jim, Sam and Bill were playing in the garden and one of them broke a garden pot. While enquiring about it the three friends answered as follows:

Jim: I haven't broken the pot.
Sam: I haven't broken the pot.
Bill: Sam has broken the pot.

<u>If exactly one of them is speaking the truth,</u> then who broke the pot?

Obviously, Jim broke the pot. <u>You must show how a machine would reach this conclusion.</u>

First, establish the ontology: Symbol J/S/B means that Jim/Sam/Bill broke the pot.
Next, reduce the statements to propositional logic (in infix form):

If <u>Jim, and only Jim,</u> is telling the truth: $((\neg J) \wedge S \wedge (\neg S))$
If <u>Sam, and only Sam,</u> is telling the truth: $(J \wedge (\neg S) \wedge (\neg S))$
If <u>Bill, and only Bill,</u> is telling the truth: $(J \wedge S \wedge S)$

Each of the three statements above reflects that only one person is telling the truth, and the others are not. <u>Exactly one</u> is telling the truth, so we have the disjunction of the three statements above:

$((\neg J) \wedge S \wedge (\neg S)) \vee (J \wedge (\neg S) \wedge (\neg S)) \vee (J \wedge S \wedge S)$

You convert this sentence into CNF to yield your Knowledge Base KB (in clausal form):

(J (¬ S)) (J S)

(Recall that your KB is an AND of its clauses, and each clause is an OR of its literals.)

Your query sentence is, "Jim broke the pot." In propositional logic form, this is "J." You negate the goal, yielding (¬ J), and add it to KB. The resulting starting point for your resolution proof is:

(J (¬ S)) (J S) (¬ J)

<u>Write a resolution proof that Jim broke the garden pot.</u>

For each step of the proof, fill in the first two blanks with CNF sentences from KB that will resolve to produce the CNF result that you write in the third (resolvent) blank. The resolvent is the result of resolving the first two sentences. Add the resolvent to KB, and repeat. Use as many steps as necessary, ending with the empty clause. The empty clause indicates a contradiction, and therefore that KB entails the original goal sentence.

The shortest proof that I know of is only two lines long. Longer proofs are OK if correct.

Resolve <u>(J (¬ S))</u> with <u>(J S)</u> to produce: <u>(J)</u>

Resolve <u>(J)</u> with <u>(¬ J)</u> to produce: <u>( )</u>

Resolve __________ with __________ to produce: __________

Resolve __________ with __________ to produce: __________

Resolve __________ with __________ to produce: __________

It is OK if you omitted the parentheses.

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A three-line proof is:

Resolve ______(J (¬ S))______ with __(¬ J)______________ to produce: __(¬ S)______________

Resolve ______(J S)__________ with __(¬ J)______________ to produce: __(S)________________

Resolve ______(¬ S)__________ with __(S)________________ to produce: __( )________________

2. (20 pts total, 10 pts each) PROBABILITY. Use the joint distribution below for the Boolean variables X, Y, and Z, to calculate the following probabilities. Your final answer should be a number in the interval [0,1]. **Show your work. Correct answer + no work = no credit. Correct answer + correct work = full credit.**

X	Y	Z	P(X, Y, Z)
t	t	t	0.16
t	t	f	0.09
t	f	t	0.03
t	f	f	0.02
f	t	t	0.15
f	t	f	0.25
f	f	t	0.20
f	f	f	0.10

2.a. (10 pts) P(Y=t) = P(X=t, Y=t, Z=t) + P(X=t, Y=t, Z=f) + P(X=f, Y=t, Z=t) + P(X=f, Y=t, Z=f)

= 0.16 + 0.09 + 0.15 + 0.25

= 0.65

2.b. (10 pts) P(Y=t | X=t) = P(X=t AND Y=t) / P(X=t)

= {P(X=t, Y=t, Z=t) + P(X=t, Y=t, Z=f)}
/ {P(X=t, Y=t, Z=t) + P(X=t, Y=t, Z=f) + P(X=t, Y=f, Z=t)
+ P(X=t, Y=f, Z=f)}

= (0.16 + 0.09) / (0.16 + 0.09 + 0.03 + 0.02) = 0.25 / 0.3 = 5/6
= 0.8333...

3. (40 pts total, 10 pts each) PROBABILITY FORMULAS. Write out the following probability formulas. Below, "in terms of X" means X should appear in your answer. All answers should be formulas, not text.

3.a. (10 pts) Write the formula for the conditional probability P(A | B).

P(A | B) = $P(A \wedge B) / P(B)$

3.b. (10 pts) Factor $P(A \wedge B \wedge C)$ completely using the Product Rule (or Chain Rule). You may use any variable ordering you wish.

$P(A \wedge B \wedge C)$ = $P(A \mid B \wedge C) * P(B \mid C) * P(C)$

Other variable orderings are OK iff correct, e.g.,
$P(A \wedge B \wedge C) = P(C \mid A \wedge B) * P(B \mid A) * P(A)$
$= P(B \mid A \wedge C) * P(C \mid A) * P(A)$, etc.

3.c. (10 pts) Write Bayes' Rule (or Bayes' Theorem).

$$P(A \mid B) = P(B \mid A) * P(A) / P(B) = \frac{P(A \mid B) * P(A)}{\sum_{a \in A} P(B \mid a) * P(a)} = \frac{P(A \mid B) * P(A)}{P(B \mid a) * P(a) + P(B \mid \neg a) * P(\neg a)}$$

Iff A is Boolean.

3.d. (10 pts) Assume A and B are conditionally independent given C. Write $P(A \wedge B \mid C)$ in terms of P(A | C) and P(B | C) and possibly other terms.

$P(A \wedge B \mid C)$ = $P(A \mid C) * P(B \mid C)$

Bayes' Rule is written in several different forms in different places, any of which gets full credit if mathematically correct.