

1. (10 points total, 5 pts off for each wrong answer, but not negative)

a. (5 pts) Write down the definition of  $P(H | D)$  in terms of  $P(H)$ ,  $P(D)$ ,  $P(H \wedge D)$ , and  $P(H \vee D)$ .

$P(H | D) =$

b. (5 pts) Write down the expression that results from applying Bayes' Rule to  $P(H | D)$ .

$P(H | D) =$

c. (5 pts) Write down the expression for  $P(H \wedge D)$  in terms of  $P(H)$ ,  $P(D)$ , and  $P(H \vee D)$ .

$P(H \wedge D) =$

d. (5 pts) Write down the expression for  $P(H \wedge D)$  in terms of  $P(H)$ ,  $P(D)$ , and  $P(H | D)$ .

$P(H \wedge D) =$

2. (10 pts total, 5 pts each) We have a database describing 100 examples of printer failures. Of these, 75 examples are hardware failures, and 25 examples are driver failures. Of the hardware failures, 15 had Windows operating system. Of the driver failures, 15 had Windows operating system. Show your work.

a. (5 pts) Calculate  $P(\text{windows} | \text{hardware})$  using the information in the problem.

b. (5 pts) Calculate  $P(\text{driver} | \text{windows})$  using Bayes' rule and the information in the problem.

3. (5 pts) After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that it is a rare disease, striking only 1 in 10,000 people of your age. What is the probability that you actually have the disease? Show your work.

4. (15 pts total, 5 pts each) Suppose you are given a bag containing  $n$  unbiased coins. You are told that  $n - 1$  of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides. Show your work for the questions below.

a. Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the conditional probability that the coin you chose is the fake coin?

b. Suppose you continue flipping the coin for a total of  $k$  times after picking it and see  $k$  heads. Now what is the conditional probability that you picked the fake coin?

c. Suppose you wanted to decide whether a chosen coin was fake by flipping it  $k$  times. The decision procedure returns FAKE if all  $k$  flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error on coins drawn from the bag?