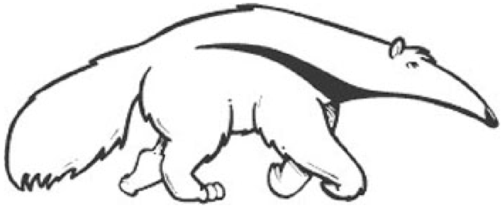
# Propositional Logic A: Syntax & Semantics

CS171, Fall Quarter, 2018
Introduction to Artificial Intelligence
Prof. Richard Lathrop



**Read Beforehand:** **R&N 7.1-7.5**
**Optional: R&N 7.6-7.8)**

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# You will be expected to know:

- Basic definitions (section 7.1, 7.3)
- Models and entailment (7.3)
- Syntax, logical connectives (7.4.1)
- Semantics (7.4.2)
- Simple inference (7.4.4)

# Complete architectures for intelligence?

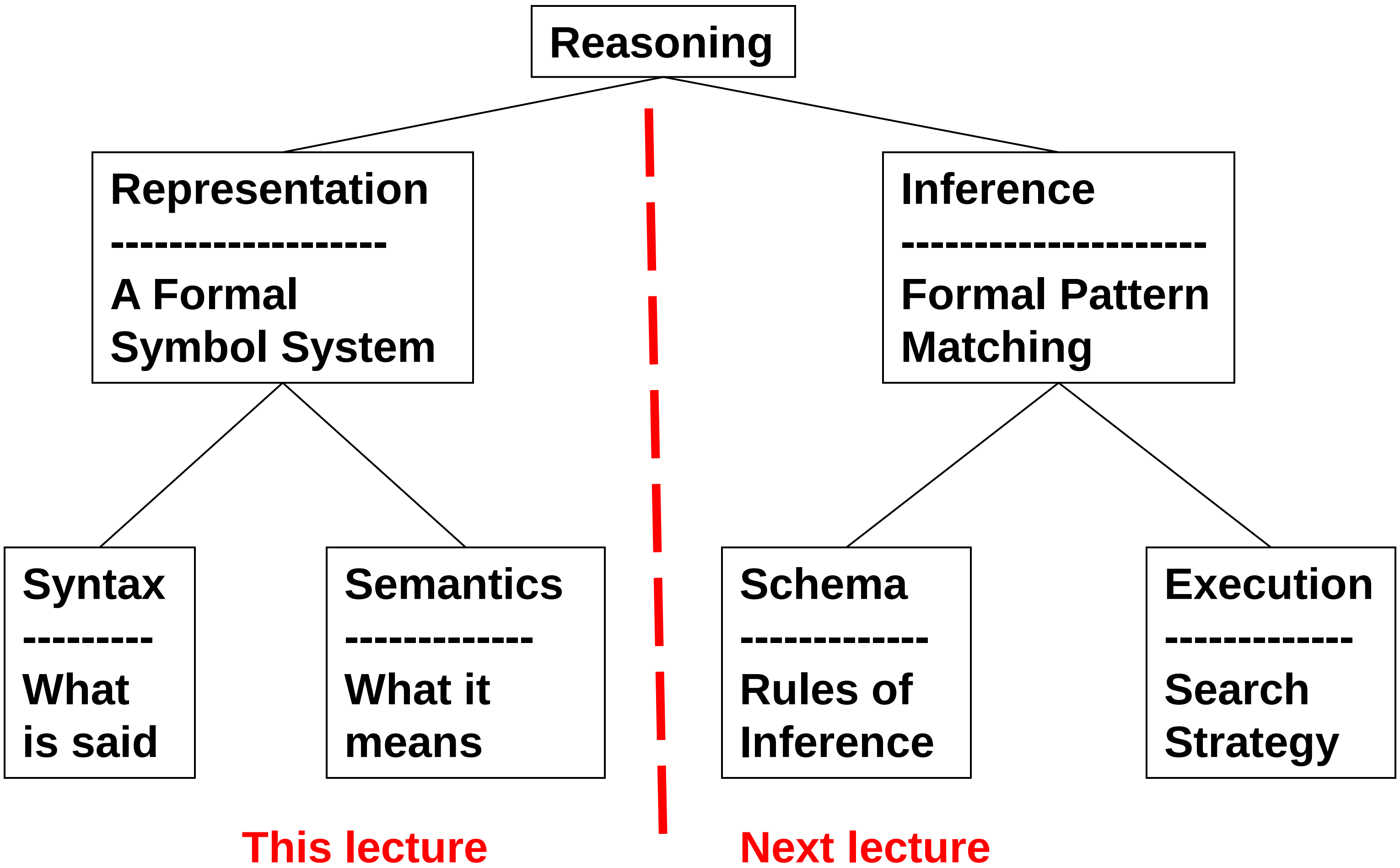
- Search?
  - Solve the problem of what to do.
- Logic and inference?
  - Reason about what to do.
  - Encoded knowledge/“expert” systems?
    - Know what to do.
- Learning?
  - Learn what to do.
- Modern view: It’s complex & multi-faceted.

# Inference in Formal Symbol Systems: Ontology, Representation, Inference

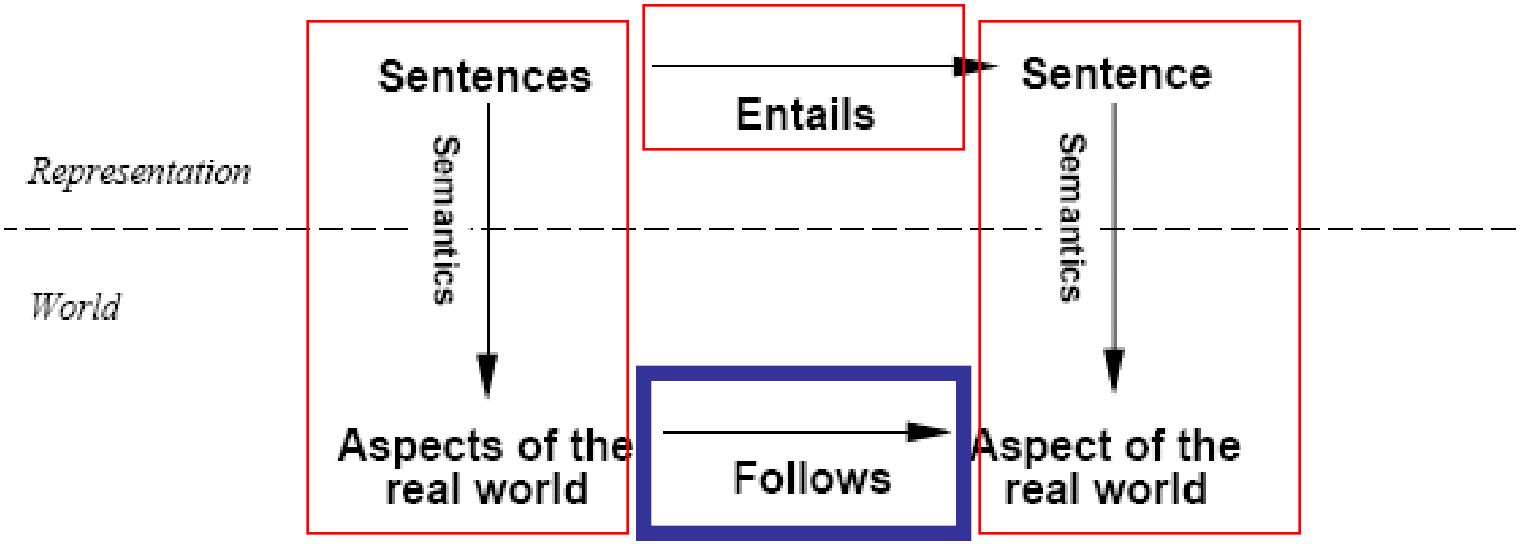
- **Formal Symbol Systems**
  - **Symbols** correspond to **things/ideas** in the world
  - **Pattern matching & rewrite** corresponds to **inference**

- **Ontology:** What exists in the world?
  - What must be represented?
- **Representation:** Syntax vs. Semantics
  - What’s Said vs. What’s Meant
- **Inference:** Schema vs. Mechanism
  - Proof Steps vs. Search Strategy

**Ontology:**
**What kind of things exist in the world?**
**What do we need to describe and reason about?**



# Schematic perspective




***If KB is true in the real world,***

***then any sentence $\alpha$ entailed by KB***

***is also true in the real world.***

For example: If I tell you (1) Sue is Mary's sister, and (2) Sue is Amy's mother, then it **necessarily follows in the world** that Mary is Amy's aunt, even though I told you nothing at all about aunts. This sort of reasoning pattern is what we hope to capture.

# Why Do We Need Logic?

- Problem-solving agents were very inflexible: hard code every possible state.

- Search is almost always exponential in the number of states.

- Problem solving agents cannot infer unobserved information.

- We want an algorithm that reasons in a way that resembles reasoning in humans.

# Knowledge-Based Agents

- **KB = knowledge base**
  - A set of sentences or facts
  - e.g., a set of statements in a logic language

- **Inference**
  - Deriving new sentences from old
  - e.g., using a set of logical statements to infer new ones

- **A simple model for reasoning**
  - Agent is told or perceives new evidence
    - E.g., agent is told or perceives that A is true
  - Agent then infers new facts to add to the KB
    - E.g., KB = { (A -> (B OR C) ); (not C) }

      then given A and not C the agent can infer that B is true
    - B is now added to the KB even though it was not explicitly asserted, i.e., the agent inferred B

# Types of Logics

- **Propositional logic:** concrete statements that are either true or false
  - E.g., John is married to Sue.
- **Predicate logic (also called first order logic, first order predicate calculus):** allows statements to contain variables, functions, and quantifiers
  - For all X, Y: If X is married to Y then Y is married to X.
- **Probability:** statements that are possibly true; the chance I win the lottery?
- **Fuzzy logic:** vague statements; paint is <u>slightly grey</u>; sky is <u>very cloudy</u>.
- **Modal logic** is a class of various logics that introduce modalities:
  - **Temporal logic:** statements about time; John was a student at UCI for <u>four years</u>, and <u>before that</u> he spent <u>six years</u> in the US Marine Corps.
  - **Belief and knowledge:** Mary <u>knows</u> that John is married to Sue; a poker player <u>believes</u> that another player will fold upon a large bluff.
  - **Possibility and Necessity:** What <u>might</u> happen (possibility) and <u>must</u> happen (necessity); I <u>might</u> go to the movies; I <u>must</u> die and pay taxes.
  - **Obligation and Permission:** It is <u>obligatory</u> that students study for their tests; it is <u>permissible</u> that I go fishing when I am on vacation.

# Other Reasoning Systems

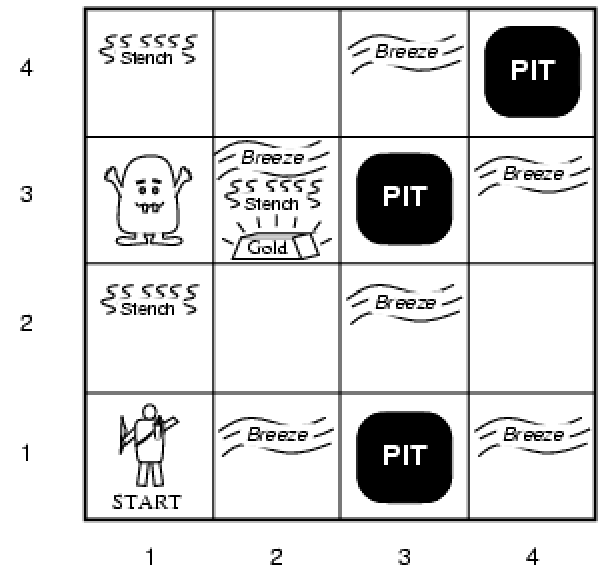
- How to produce new facts from old facts?
- **Induction**
  - Reason from facts to the general law
  - Scientific reasoning, machine learning
- **Abduction**
  - Reason from facts to the best explanation
  - Medical diagnosis, hardware debugging
- **Analogy (and metaphor, simile)**
  - Reason that a new situation is like an old one

# Wumpus World PEAS description

- Performance measure
  - gold: +1000, death: -1000
  - -1 per step, -10 for using the arrow

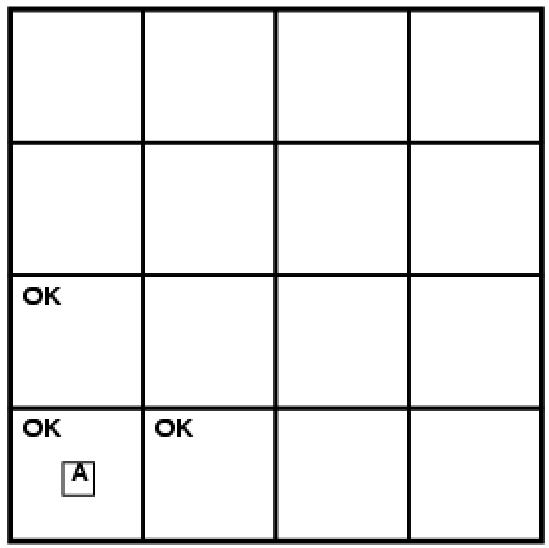
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square

**Would DFS work well? A*?**




- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

# Exploring a wumpus world



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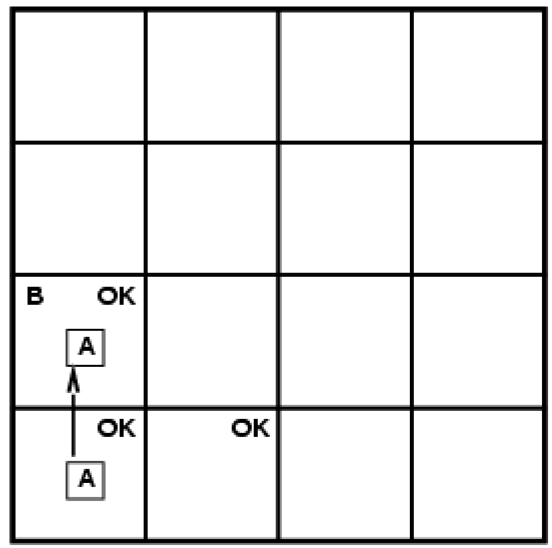

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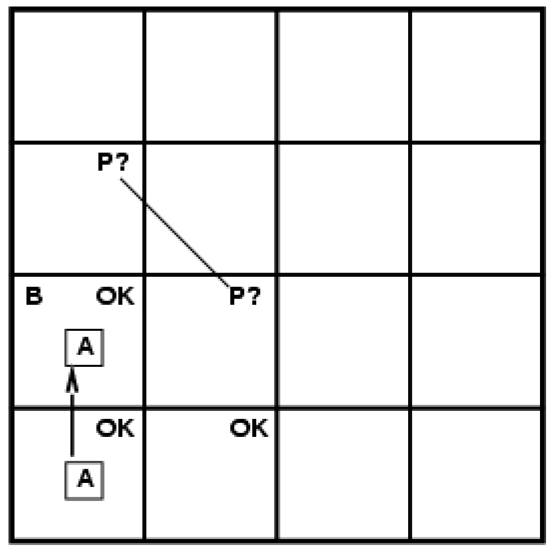

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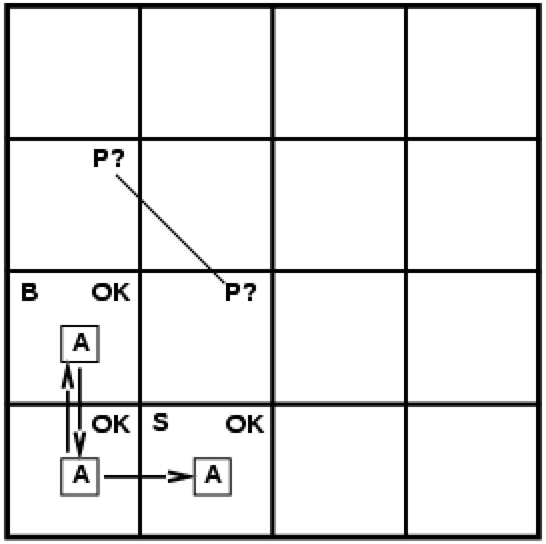

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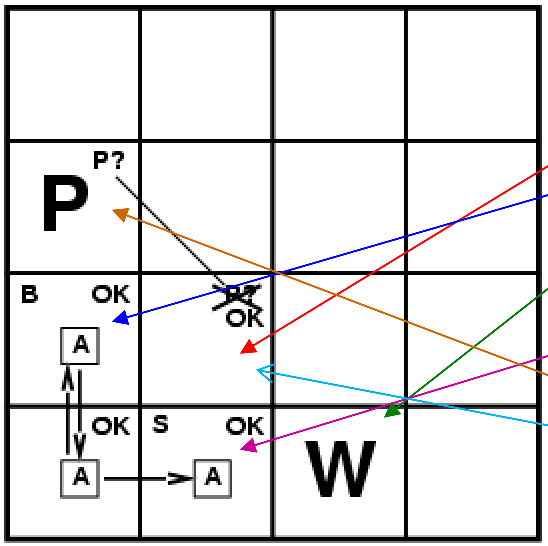
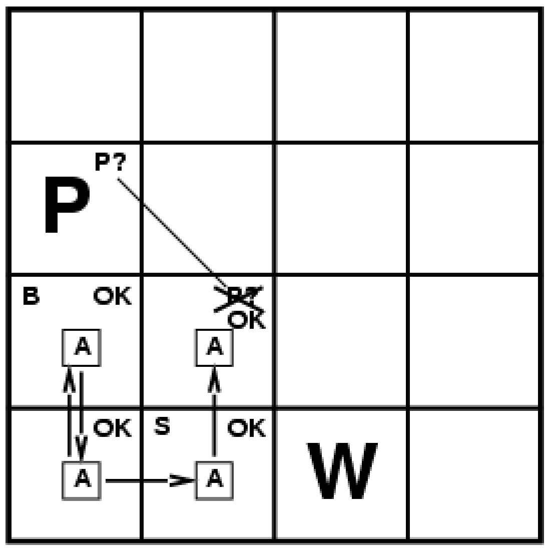



If the Wumpus were here, stench should be here. Therefore it is here.
Since, there is no breeze here, the pit must be there, and it must be OK here

**We need rather sophisticated reasoning here!**

# Exploring a wumpus world



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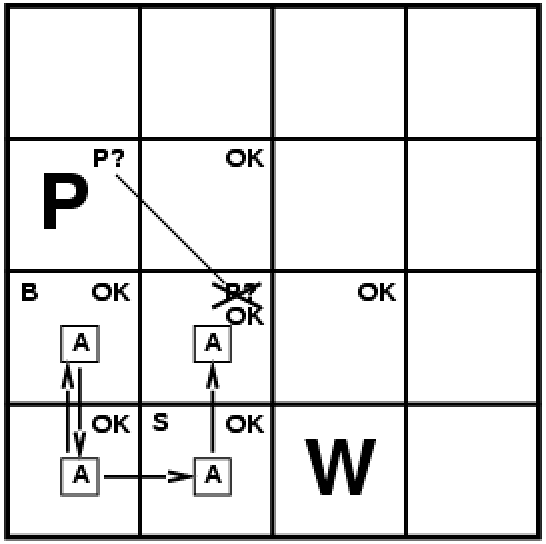

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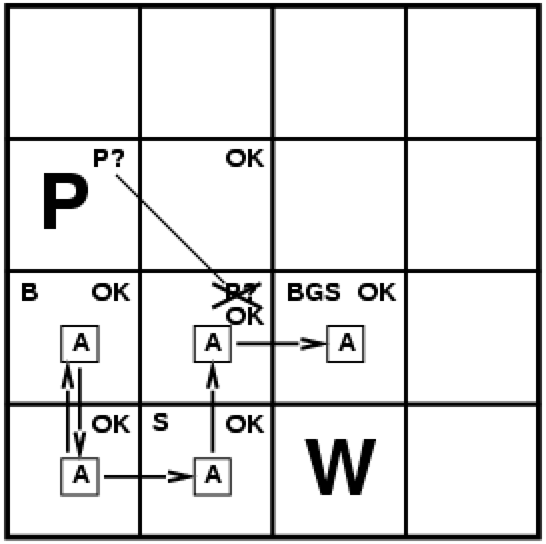


# Logic

- We used logical reasoning to find the gold.
- Logics are formal languages for representing information such that conclusions can be drawn from formal inference patterns
- Syntax defines the well-formed sentences in the language
- Semantics define the "meaning" or interpretation of sentences:
  - connect symbols to real events in the world
  - i.e., define truth of a sentence in a world

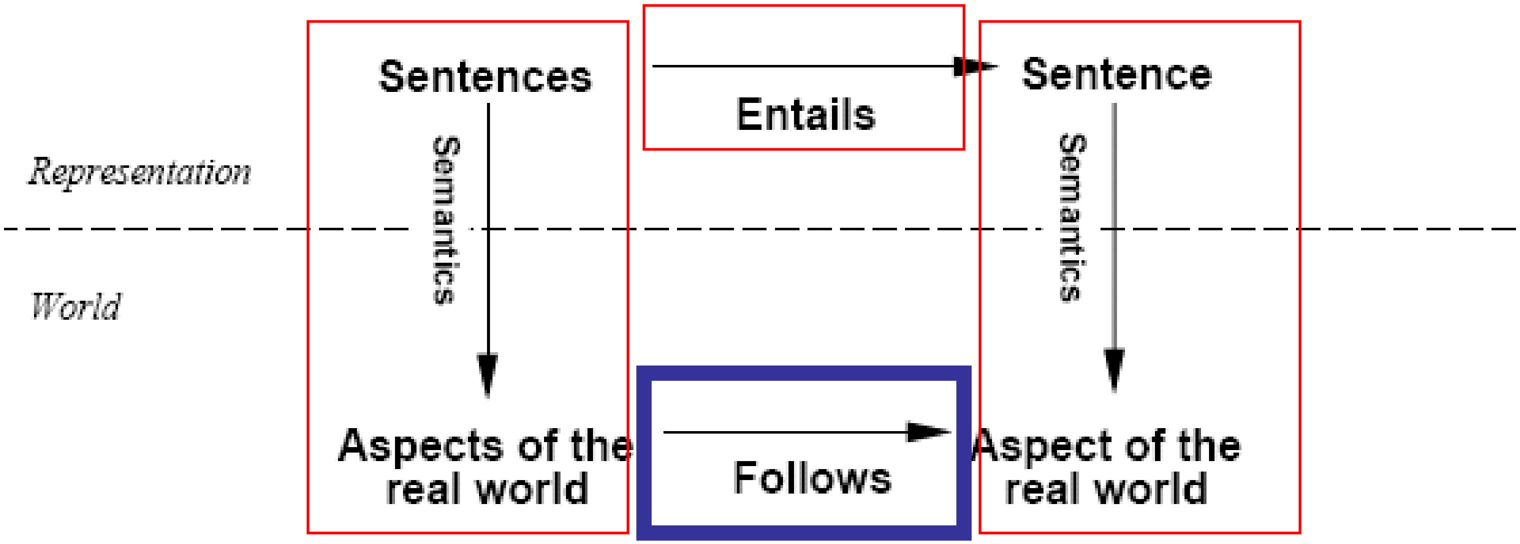
- E.g., the language of arithmetic:
  - $x+2 \geq y$ is a sentence
  - x2+y > {} is not a sentence

  → syntax

  - $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0$, $y = 6$

  → semantics

# Schematic perspective

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For example: If I tell you (1) Sue is Mary's sister, and (2) Sue is Amy's mother, then it **necessarily follows in the world** that Mary is Amy's aunt, even though I told you nothing at all about aunts. This sort of reasoning pattern is what we hope to capture.

# Entailment

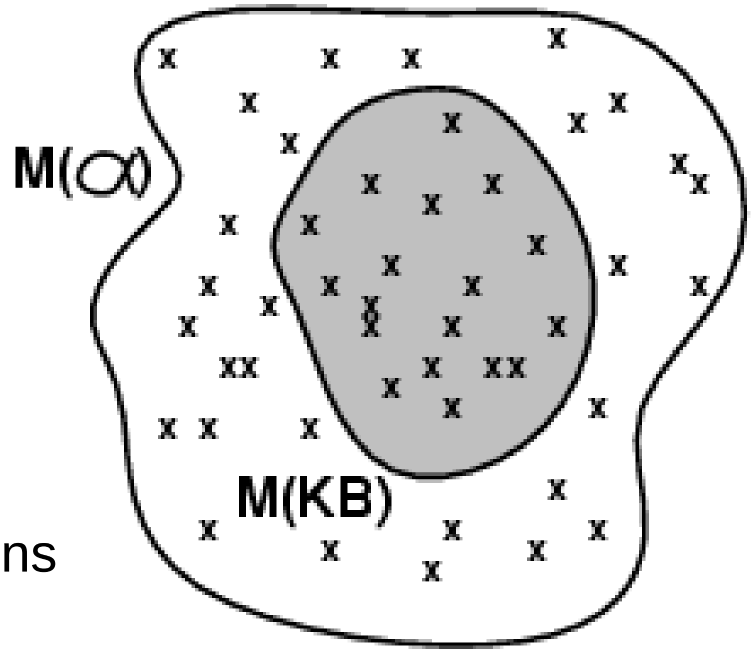
- Entailment means that one thing follows from another set of things:

$$KB \models \alpha$$

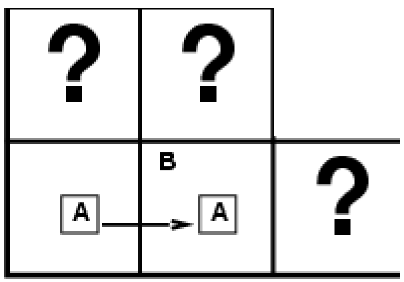
- Knowledge base *KB* entails sentence α if and only if α is true in all worlds wherein *KB* is true
  - E.g., the KB = "the Giants won and the Reds won" entails α = "The Giants won".
  - E.g., KB = "x+y = 4" entails α = "4 = x+y"
  - E.g., KB = "Mary is Sue's sister and Amy is Sue's daughter" entails α = "Mary is Amy's aunt."
- The entailed α MUST BE TRUE in ANY world in which KB IS TRUE.

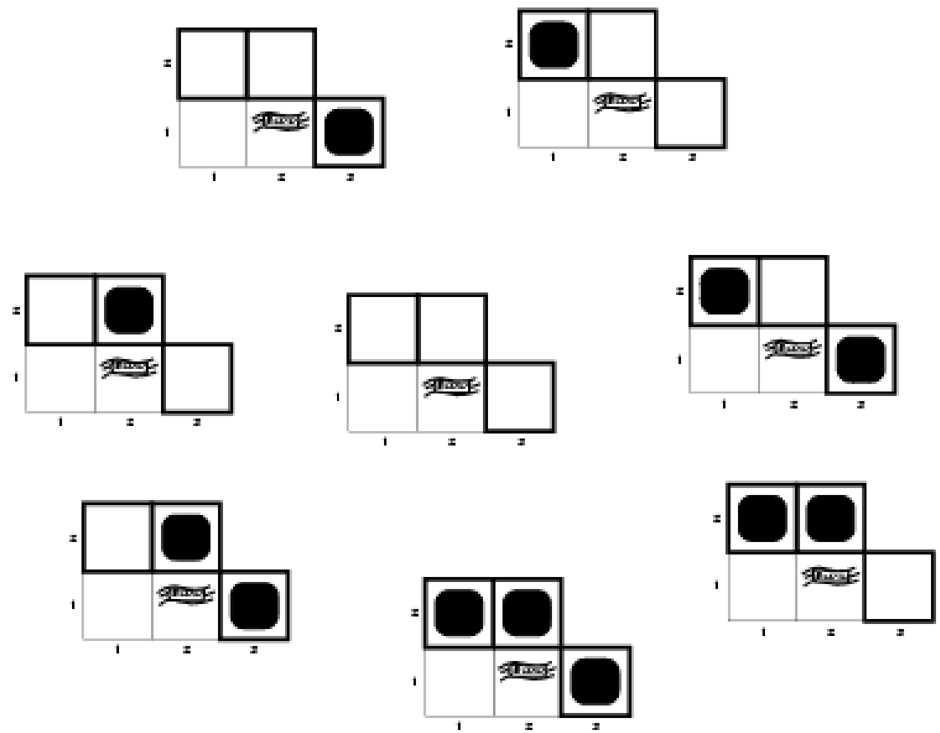
# Models

- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then KB $\models$ α iff *M(KB)* $\subseteq$ *M*(α)
  - E.g. *KB* = Giants won and Reds won entails α = Giants won
- Think of KB and α as collections of constraints and of models m as possible states. M(KB) are the solutions to KB and M(α) the solutions to α. Then, KB $\models$ α when all solutions to KB are also solutions to α.



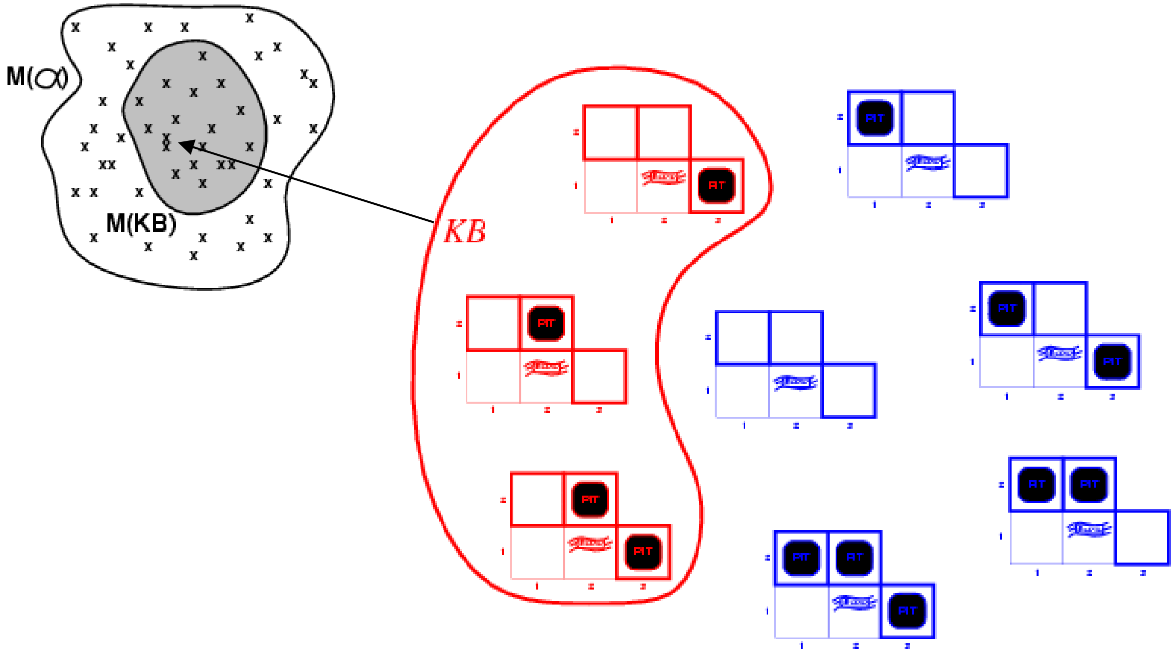
# Wumpus models



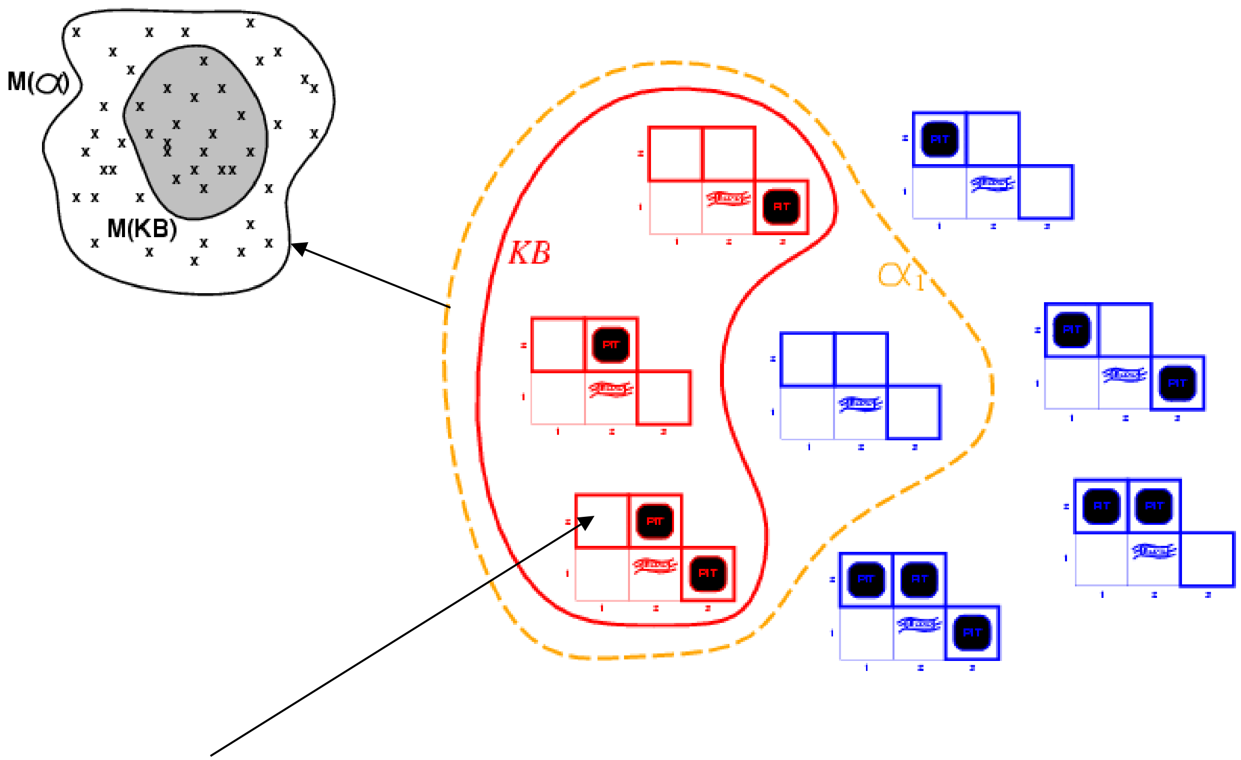
**All possible models in this reduced Wumpus world. What can we infer?**

# Wumpus models




- *M(KB)* = all possible wumpus-worlds consistent with the observations and the "physics" of the Wumpus world.

# Wumpus models



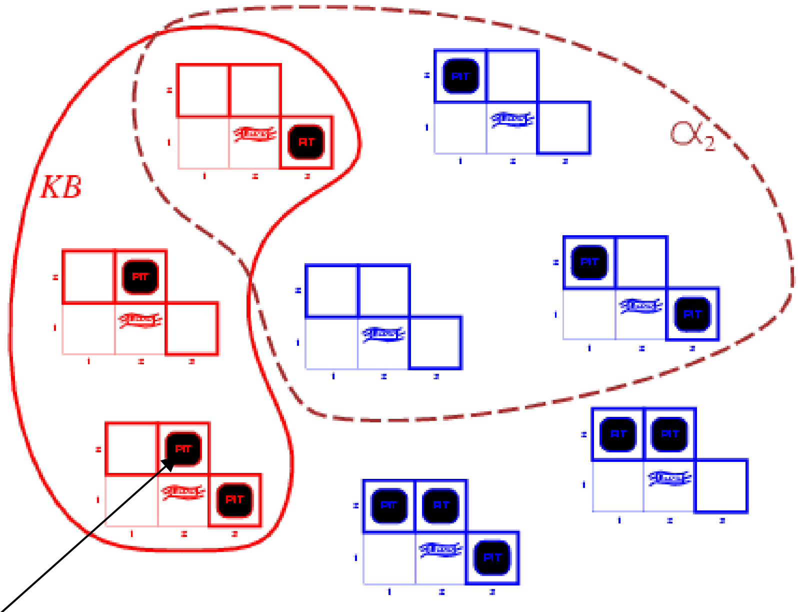

Now we have a query sentence, $\alpha_1$ = "[1,2] is safe"

*KB* $\models \alpha_1$, proved by **model checking**

M(KB) (red outline) is a subset of M($\alpha_1$) (orange dashed outline)

$\Rightarrow$ **$\alpha_1$ is true in any world in which KB is true**

# Wumpus models




Now we have another query sentence, $\alpha_2$ = "[2,2] is safe"

*KB* $\not\models$ $\alpha_2$, proved by **model checking**

M(KB) (red outline) is a **not** a subset of M($\alpha_2$) (dashed outline)

**$\Rightarrow$ $\alpha_2$ is false in some world(s) in which KB is true**

# Recap propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas
- The proposition symbols $P_1$, $P_2$ etc are sentences
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If $S_1$ and $S_2$ are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If $S_1$ and $S_2$ are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If $S_1$ and $S_2$ are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If $S_1$ and $S_2$ are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

# Recap propositional logic: Semantics

Each model/world specifies true or false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
false true false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

| | | | |
|---|---|---|---|
| $\neg S$ | is true iff* | $S$ is false | |
| $S_1 \wedge S_2$ | is true iff | $S_1$ is true and | $S_2$ is true |
| $S_1 \vee S_2$ | is true iff | $S_1$ is true or | $S_2$ is true |
| $S_1 \Rightarrow S_2$ | is true iff | $S_1$ is false or | $S_2$ is true |
| i.e., | is false iff | $S_1$ is true and | $S_2$ is false |
| $S_1 \Leftrightarrow S_2$ | is true iff | $S_1 \Rightarrow S_2$ is true and | $S_2 \Rightarrow S_1$ is true |

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$$

* iff = if and only if

# Recap truth tables for connectives

| $P$ | $Q$ | $\neg P$ | $P \wedge Q$ | $P \vee Q$ | $P \Rightarrow Q$ | $P \Leftrightarrow Q$ |
|---|---|---|---|---|---|---|
| $false$ | $false$ | $true$ | $false$ | $false$ | $true$ | $true$ |
| $false$ | $true$ | $true$ | $false$ | $true$ | $true$ | $false$ |
| $true$ | $false$ | $false$ | $false$ | $true$ | $false$ | $false$ |
| $true$ | $true$ | $false$ | $true$ | $true$ | $true$ | $true$ |

**OR: P or Q is true or both are true.**
**XOR: P or Q is true but not both.**

**Implication is always true when the premises are False!**

# Inference by enumeration
## (generate the truth table = model checking)

- Enumeration of all models is sound and complete.
- For *n* symbols, time complexity is $O(2^n)$...
- We need a smarter way to do inference!
- In particular, we are going to infer new logical sentences from the data-base and see if they match a query.

# Logical equivalence

- To manipulate logical sentences we need some rewrite rules.
- Two sentences are logically equivalent iff they are true in same models: $\alpha \equiv$ ß iff $\alpha \models \beta$ and $\beta \models \alpha$

**You need to know these !**

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \lor \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{de Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

# Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \lor B$, $C$

A sentence is unsatisfiable if it is false in all models

e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg \alpha)$ is unsatisfiable

(there is no model for which KB=true and $\alpha$ is false)

# Summary (Part I)

- Logical agents apply inference to a knowledge base to derive new information and make decisions

- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
  - valid: sentence is true in every model (a tautology)

- Logical equivalences allow syntactic manipulations

- Propositional logic lacks expressive power
  - Can only state specific facts about the world.
  - Cannot express general rules about the world (use First Order Predicate Logic)