First-Order Logic A: Syntax

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Introduction to Artificial Intelligence
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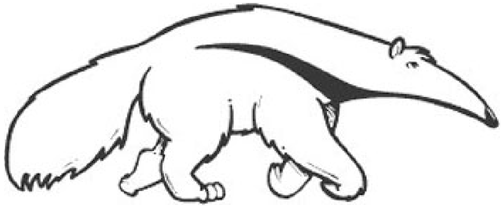

Read Beforehand: R&N 8, 9.1-9.2, 9.5.1-9.5.5

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Common Sense Reasoning
Example, adapted from Lenat

You are told: John drove to the grocery store and bought a pound of noodles, a pound of ground beef, and two pounds of tomatoes.

- Is John 3 years old?
- Is John a child?
- What will John do with the purchases?
- Did John have any money?
- Does John have less money after going to the store?
- Did John buy at least two tomatoes?
- Were the tomatoes made in the supermarket?
- Did John buy any meat?
- Is John a vegetarian?
- Will the tomatoes fit in John’s car?

- Can Propositional Logic support these inferences?

Outline for First-Order Logic (FOL, also called FOPC)

- Propositional Logic is **Useful** --- but **Limited Expressive Power**
- First Order Predicate Calculus (FOPC), or First Order Logic (FOL).
  - FOPC has expanded expressive power, though still limited.
- New Ontology
  - The world consists of OBJECTS.
  - OBJECTS have PROPERTIES, RELATIONS, and FUNCTIONS.
- New Syntax
  - Constants, Predicates, Functions, Properties, Quantifiers.
- New Semantics
  - Meaning of new syntax.
- Unification and Inference in FOL
- Knowledge engineering in FOL

FOL Syntax:
You will be expected to know

- FOPC syntax
  - Syntax: Sentences, predicate symbols, function symbols, constant symbols, variables, quantifiers
- De Morgan's rules for quantifiers
  - connections between $\forall$ and $\exists$
- Nested quantifiers
  - Difference between "$\forall$ x $\exists$ y P(x, y)" and "$\exists$ x $\forall$ y P(x, y)"
  - $\forall$ x $\exists$ y Likes(x, y) --- "Everybody likes somebody."
  - $\exists$ x $\forall$ y Likes(x, y) --- "Somebody likes everybody."
- Translate simple English sentences to FOPC and back
  - $\forall$ x $\exists$ y Likes(x, y) $\Leftrightarrow$ "Everyone has someone that they like."
  - $\exists$ x $\forall$ y Likes(x, y) $\Leftrightarrow$ "There is someone who likes every person."

Pros and cons of propositional logic

- ☺ Propositional logic is declarative
  - Knowledge and inference are separate
- ☺ Propositional logic allows partial/disjunctive/negated information
  - unlike most programming languages and databases
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - unlike natural language, where meaning depends on context
- ☹ Propositional logic has **limited expressive power**
  - E.g., cannot say "Pits cause breezes in adjacent squares."
    - except by writing one sentence for each square
  - Needs to refer to objects in the world,
  - Needs to express general rules

First-Order Logic (FOL), also called
First-Order Predicate Calculus (FOPC)

- Propositional logic assumes the world contains facts.

- First-order logic (like natural language) assumes the world contains

  - **Objects**: people, houses, numbers, colors, baseball games, wars, …
  - **Functions**: father of, best friend, one more than, plus, …
    - Function arguments are objects; function returns an object
  - **Objects generally correspond to English NOUNS**

  - **Predicates/Relations/Properties**: red, round, prime, brother of, bigger than, part of, comes between, …
    - Predicate arguments are objects; predicate returns a truth value
  - **Predicates generally correspond to English VERBS**
    - **First argument is generally the subject, the second the object**
    - Hit(Bill, Ball) usually means “Bill hit the ball.”
    - Likes(Bill, IceCream) usually means “Bill likes IceCream.”
    - Verb(Noun1, Noun2) usually means “Noun1 verb noun2.”

Aside: First-Order Logic (FOL) vs. Second-Order Logic

- First Order Logic (FOL) allows variables and general rules
  - "First order" because quantified variables represent objects.
  - "Predicate Calculus" because it quantifies over predicates on objects.
    - E.g., "Integral Calculus" quantifies over functions on numbers.
- Aside: Second Order logic
  - "Second order" because quantified variables can also represent predicates and functions.
    - E.g., can define "Transitive Relation," which is beyond FOPC.

- Aside: In FOL we can state that a relationship is transitive
  - E.g., BrotherOf is a transitive relationship
  - $\forall$ x, y, z BrotherOf(x,y) $\wedge$ BrotherOf(y,z) => BrotherOf(x,z)
- Aside: In Second Order logic we can define "Transitive"
  - $\forall$ P, x, y, z Transitive(P) $\Leftrightarrow$ (P(x,y) $\wedge$ P(y,z) => P(x,z))
  - Then we can state directly, Transitive(BrotherOf)

Syntax of FOL: Basic elements

• Constants	KingJohn, 2, UCI,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Quantifiers	$\forall$, $\exists$
• Connectives	$\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ (standard)
• Equality	= (but causes difficulties....)

Syntax of FOL: Basic syntax elements are symbols

- **Constant** Symbols (correspond to English nouns)
  - Stand for objects in the world.
    - E.g., KingJohn, 2, UCI, ...
- **Predicate** Symbols (correspond to English verbs)
  - Stand for relations (maps a tuple of objects to a **truth-value**)
    - E.g., Brother(Richard, John), greater_than(3,2), ...
  - P(x, y) is usually read as "x is P of y."
    - E.g., Mother(Ann, Sue) is usually "Ann is Mother of Sue."
- **Function** Symbols (correspond to English nouns)
  - Stand for functions (maps a tuple of objects to an **object**)
    - E.g., Sqrt(3), LeftLegOf(John), ...
- **Model** (world) = set of domain objects, relations, functions
- **Interpretation** maps symbols onto the model (world)
  - Very many interpretations are possible for each KB and world!
  - The KB is to rule out those inconsistent with our knowledge.

Syntax of FOL: Terms

- **Term** = logical expression that **refers to an object**
- **There are two kinds of terms:**
  - **Constant Symbols** stand for (or name) objects:
    - E.g., KingJohn, 2, UCI, Wumpus, ...
  - **Function Symbols** map tuples of objects to an object:
    - E.g., LeftLeg(KingJohn), Mother(Mary), Sqrt(x)
    - This is nothing but a complicated kind of name
      - No “subroutine” call, no “return value”

Syntax of FOL: Atomic Sentences

- **Atomic Sentences** state facts (logical truth values).
  - An **atomic sentence** is a Predicate symbol, optionally followed by a parenthesized list of any argument terms
    - E.g., *Married(Father(Richard), Mother(John))*
  - An **atomic sentence** asserts that some relationship (some predicate) holds among the objects that are its arguments.

- An **Atomic Sentence is true** in a given model if the relation referred to by the predicate symbol holds among the objects (terms) referred to by the arguments.

Syntax of FOL: Atomic Sentences

- Atomic sentences in logic state facts that are true or false.

- Properties and *m*-ary relations do just that:

  LargerThan(2, 3) is false.

  BrotherOf(Mary, Pete) is false.

  Married(Father(Richard), Mother(John)) could be true or false.

  Properties and *m*-ary relations are Predicates that are true or false.

- Note: Functions refer to objects, do not state facts, and form no sentence:
  - Brother(Pete) refers to John (his brother) and is neither true nor false.
  - Plus(2, 3) refers to the number 5 and is neither true nor false.

- BrotherOf(Pete, Brother(Pete)) is True.

Binary relation
is a truth value.

Function refers to John, an object in the
world, i.e., John is Pete's brother.
(Works well iff John is Pete's only brother.)

Syntax of FOL: Connectives & Complex Sentences

- **Complex Sentences** are formed in the same way, using the same logical connectives, as in propositional logic

- The **Logical Connectives**:
  - $\Leftrightarrow$ biconditional
  - $\Rightarrow$ implication
  - $\wedge$ and
  - $\vee$ or
  - $\neg$ negation

- **Semantics** for these logical connectives are the same as we already know from propositional logic.

Examples

- $Brother(Richard, John) \wedge Brother(John, Richard)$

- $King(Richard) \vee King(John)$

- $King(John) \Rightarrow \neg King(Richard)$

- $LessThan(Plus(1,2), 4) \wedge GreaterThan(1,2)$

Syntax of FOL: Variables

- **Variables** range over objects in the world.
- A **variable** is like a **term** because it represents an object.
- A **variable** may be used wherever a **term** may be used.
  - **Variables** may be arguments to functions and predicates.
- (A **term with NO variables** is called a **ground term**.)
- (A **variable not bound by a quantifier** is called **free**.)
  - All variables we will use are bound by a quantifier.

Syntax of FOL: Logical Quantifiers

- There are two **Logical Quantifiers:**
  - **Universal:** $\forall$ x P(x) means "For all x, P(x)."
    - The "upside-down A" reminds you of "ALL."
    - Some texts put a comma after the variable: $\forall$ x, P(x)
  - **Existential:** $\exists$ x P(x) means "There exists x such that, P(x)."
    - The "backward E" reminds you of "EXISTS."
    - Some texts put a comma after the variable: $\exists$ x, P(x)

- You can ALWAYS convert one quantifier to the other.
  - $\forall x\, P(x) \equiv \neg\exists x\, \neg P(x)$
  - $\exists x\, P(x) \equiv \neg\forall x\, \neg P(x)$
  - **RULES:** $\forall \equiv \neg\exists\neg$ and $\exists \equiv \neg\forall\neg$

- **RULES:** To move negation "in" across a quantifier,

  Change the quantifier to "the other quantifier"
  and negate the predicate on "the other side."
  - $\neg\forall x\, P(x) \equiv \neg\, \neg\exists x\, \neg P(x) \equiv \exists x\, \neg P(x)$
  - $\neg\exists x\, P(x) \equiv \neg\, \neg\forall x\, \neg P(x) \equiv \forall x\, \neg P(x)$

Universal Quantification $\forall$

- $\forall$ x means "for all x it is true that…"

- Allows us to make statements about all objects that have certain properties

- Can now state general rules:

  $\forall$ x King(x) => Person(x) "All kings are persons."
  $\forall$ x Person(x) => HasHead(x) "Every person has a head."
  $\forall$ i Integer(i) => Integer(plus(i,1)) "If i is an integer then i+1 is an integer."

- **Note: $\forall$ x King(x) $\wedge$ Person(x) is not correct!**

  This would imply that all objects x are Kings and are People (!)

  $\forall$ x King(x) => Person(x) is the correct way to say this

- **Note that => is the natural connective to use with $\forall$.**

Universal Quantification $\forall$

- Universal quantification is <u>conceptually</u> equivalent to:
  - Conjunction of all sentences obtained by substitution of an object for the quantified variable.
  - <u>Not a sentence in the logic</u> --- all logic sentences must be finite.

- Example: All Cats are Mammals.
  - $\forall$x Cat(x) $\Rightarrow$ Mammal(x)

- Conjunction of all sentences obtained by substitution of an object for the quantified variable:

  Cat(Spot) $\Rightarrow$ Mammal(Spot) $\wedge$
  Cat(Rebecca) $\Rightarrow$ Mammal(Rebecca) $\wedge$
  Cat(LAX) $\Rightarrow$ Mammal(LAX) $\wedge$
  Cat(Shayama) $\Rightarrow$ Mammal(Shayama) $\wedge$
  Cat(France) $\Rightarrow$ Mammal(France) $\wedge$
  Cat(Felix) $\Rightarrow$ Mammal(Felix) $\wedge$
  …

Existential Quantification $\exists$

- $\exists$ x means "there exists an x such that…."
  - There is in the world at least one such object x

- Allows us to make statements about some object without naming it, or even knowing what that object is:

  $\exists$ x King(x) "Some object is a king."
  $\exists$ x Lives_in(John, Castle(x)) "John lives in somebody's castle."
  $\exists$ i Integer(i) $\wedge$ Greater(i,0) "Some integer is greater than zero."

- **Note: $\exists$ i Integer(i) $\Rightarrow$ Greater(i,0) is not correct!**

  It is vacuously true if anything in the world were not an integer (!)

  $\exists$ i Integer(i) $\wedge$ Greater(i,0) is the correct way to say this

- **Note that $\wedge$ is the natural connective to use with $\exists$.**

Existential Quantification $\exists$

- Existential quantification is <u>conceptually</u> equivalent to:
  - Disjunction of all sentences obtained by substitution of an object for the quantified variable.
  - Not a sentence in the logic --- all logic sentences must be finite.

- Spot has a sister who is a cat.
  - $\exists x$ Sister(x, Spot) $\wedge$ Cat(x)

- Disjunction of all sentences obtained by substitution of an object for the quantified variable:

  Sister(Spot, Spot) $\wedge$ *Cat(Spot)* $\vee$
  Sister(Rebecca, Spot) $\wedge$ *Cat(Rebecca)* $\vee$
  Sister(LAX, Spot) $\wedge$ *Cat(LAX)* $\vee$
  Sister(Shayama, Spot) $\wedge$ *Cat(Shayama)* $\vee$
  Sister(France, Spot) $\wedge$ *Cat(France)* $\vee$
  Sister(Felix, Spot) $\wedge$ *Cat(Felix)* $\vee$
  …

Combining Quantifiers --- Order (Scope)

The order of "unlike" quantifiers is important.

Like nested variable scopes in a programming language.

Like nested ANDs and ORs in a logical sentence.

$\forall$ x $\exists$ y Loves(x,y)

- For everyone ("all x") there is someone ("exists y") whom they love.
- There might be a different y for each x (y is inside the scope of x)

$\exists$ y $\forall$ x Loves(x,y)

- There is someone ("exists y") whom everyone loves ("all x").
- Every x loves the same y (x is inside the scope of y)

Clearer with parentheses: $\exists$ y ($\forall$ x Loves(x,y))

The order of "like" quantifiers does not matter.

Like nested ANDs and ANDs in a logical sentence

$\forall x\ \forall y\ P(x, y) \equiv \forall y\ \forall x\ P(x, y)$

$\exists x\ \exists y\ P(x, y) \equiv \exists y\ \exists x\ P(x, y)$

Connections between Quantifiers

- Asserting that all x have property P is the same as asserting that does not exist any x that does not have the property P

$\forall$ x Likes(x, CS-171 class) $\Leftrightarrow$ $\neg \exists$ x $\neg$ Likes(x, CS-171 class)

- Asserting that there exists an x with property P is the same as asserting that not all x do not have the property P

$\exists$ x Likes(x, IceCream) $\Leftrightarrow$ $\neg \forall$ x $\neg$ Likes(x, IceCream)

In effect:

- $\forall$ is a conjunction over the universe of objects
- $\exists$ is a disjunction over the universe of objects

Thus, DeMorgan's rules can be applied

De Morgan's Law for Quantifiers

De Morgan's Rule

$$P \wedge Q \equiv \neg(\neg P \vee \neg Q)$$

$$P \vee Q \equiv \neg(\neg P \wedge \neg Q)$$

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Generalized De Morgan's Rule

$$\forall x\, P \equiv \neg \exists x\, (\neg P)$$

$$\exists x\, P \equiv \neg \forall x\, (\neg P)$$

$$\neg \forall x\, P \equiv \exists x\, (\neg P)$$

$$\neg \exists x\, P \equiv \forall x\, (\neg P)$$

AND/OR Rule is simple: if you bring a negation inside a disjunction or a conjunction, always switch between them ($\neg$ OR → AND $\neg$; $\neg$ AND → OR $\neg$).

QUANTIFIER Rule is similar: if you bring a negation inside a universal or existential, always switch between them ($\neg\, \exists \rightarrow \forall\, \neg$; $\neg\, \forall \rightarrow \exists\, \neg$).

De Morgan's Law for Quantifiers

De Morgan's Rule

$P \wedge Q \equiv \neg (\neg P \vee \neg Q)$
$P \vee Q \equiv \neg (\neg P \wedge \neg Q)$

$\neg (P \wedge Q) \equiv (\neg P \vee \neg Q)$
$\neg (P \vee Q) \equiv (\neg P \wedge \neg Q)$

Generalized De Morgan's Rule

$\forall x\, P(x) \equiv \neg \exists x \neg P(x)$
$\exists x\, P(x) \equiv \neg \forall x \neg P(x)$

$\neg \forall x\, P(x) \equiv \exists x \neg P(x)$
$\neg \exists x\, P(x) \equiv \forall x \neg P(x)$

<u>**AND/OR Rule is simple:**</u> if you bring a negation inside a disjunction or a conjunction, always switch between them ($\neg$ OR $\rightarrow$ AND $\neg$; $\neg$ AND $\rightarrow$ OR $\neg$).

<u>**QUANTIFIER Rule is similar:**</u> if you bring a negation inside a universal or existential, always switch between them ($\neg \exists \rightarrow \forall \neg$; $\neg \forall \rightarrow \exists \neg$).

Aside: More syntactic sugar --- uniqueness

- $\exists!$ x is “syntactic sugar” for “There exists a unique x”
  - “There exists one and only one x”
  - “There exists exactly one x”
  - Sometimes $\exists!$ is written as $\exists^1$

- For example, $\exists!$ x PresidentOfTheUSA(x)
  - “There is exactly one PresidentOfTheUSA.”

- This is just syntactic sugar:
  - $\exists!\ x\ P(x)$ is the same as $\exists\ x\ P(x) \wedge (\forall\ y\ P(y) \Rightarrow (x = y))$
  - “Syntactic sugar” = a convenient syntax abbreviation/extension

Equality

- $term_1 = term_2$ is true under a given interpretation
  if and only if $term_1$ and $term_2$ refer to the same object
- E.g., definition of *Sibling* in terms of *Parent*, using = is:

$$
\begin{aligned}
&\forall x,y\ Sibling(x,y) \Leftrightarrow \\
&\quad [\neg(x = y) \land \\
&\quad\ \exists m,f\ \ \neg (m = f) \land Parent(m,x) \land Parent(f,x) \\
&\qquad\qquad\qquad \land Parent(m,y) \land Parent(f,y)]
\end{aligned}
$$

- **Equality can make reasoning much more difficult!**
  - **(See R&N, section 9.5.5, page 353)**
  - You may not know when two objects are equal.
    - E.g., Ancients did not know (MorningStar = EveningStar = Venus)
  - You may have to prove x = y before proceeding
    - E.g., a resolution prover may not know 2+1 is the same as 1+2 or 4–1

Syntactic Ambiguity

- FOPC provides many ways to represent the same thing.
- E.g., “Ball-5 is red.”
  - HasColor(Ball-5, Red)
    - Ball-5 and Red are objects related by HasColor.
  - Red(Ball-5)
    - Red is a unary predicate applied to the Ball-5 object.
  - HasProperty(Ball-5, Color, Red)
    - Ball-5, Color, and Red are objects related by HasProperty.
  - ColorOf(Ball-5) = Red
    - Ball-5 and Red are objects, and ColorOf() is a function.
  - HasColor(Ball-5(), Red())
    - Ball-5() and Red() are functions of zero arguments that both return an object, which objects are related by HasColor.
  - …
- This can GREATLY confuse a pattern-matching reasoner.
  - Especially if multiple people collaborate to build the KB, and they all have different representational conventions.

Syntactic Ambiguity --- Partial Solution

- FOL can be TOO expressive, can offer TOO MANY choices
- Likely confusion, especially for **teams** of Knowledge Engineers
- Different team members can make different representation choices
  - E.g., represent "Ball43 is Red." as:
    - a predicate (= verb)? E.g., "Red(Ball43)" ?
    - an object (= noun)? E.g., "Red = Color(Ball43))" ?
    - a property (= adjective)? E.g., "HasProperty(Ball43, Red)" ?
- PARTIAL SOLUTION:
  - An upon-agreed **ontology** that settles these questions
  - Ontology = what exists in the world & how it is represented
  - The Knowledge Engineering teams agrees upon an ontology BEFORE they begin encoding knowledge

Fun with sentences

Brothers are siblings

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$

"Sibling" is symmetric

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$

One's mother is one's female parent

Fun with sentences

Brothers are siblings

$\forall x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$

"Sibling" is symmetric

$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$

One's mother is one's female parent

$\forall x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y)).$

A first cousin is a child of a parent's sibling

Fun with sentences

Brothers are siblings

$$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$$

"Sibling" is symmetric

$$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$$

One's mother is one's female parent

$$\forall\, x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y)).$$

A first cousin is a child of a parent's sibling

$$\forall\, x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists\, p, ps \;\; Parent(p, x) \land Sibling(ps, p) \land Parent(ps, y)$$

More fun with sentences

- **“All persons are mortal.”**
- [Use: Person(x), Mortal (x)]

More fun with sentences

- **“All persons are mortal.”**
  [Use: Person(x), Mortal (x)]

- $\forall x$ Person(x) $\Rightarrow$ Mortal(x)

- **Equivalent Forms:**
- $\forall x$ $\neg$Person(x) $\vee$ Mortal(x)

- **Common Mistakes:**
- $\forall x$ Person(x) $\wedge$ Mortal(x)

More fun with sentences

- **“Fifi has a sister who is a cat.”**
- [Use: Sister(Fifi, x), Cat(x)]
-

More fun with sentences

- **“Fifi has a sister who is a cat.”**
- [Use: Sister(Fifi, x), Cat(x)]
- $\exists x \; \text{Sister(Fifi, x)} \wedge \text{Cat(x)}$
- **Common Mistakes:**
- $\exists x \; \text{Sister(Fifi, x)} \Rightarrow \text{Cat(x)}$

More fun with sentences

- **“For every food, there is a person who eats that food.”**

  [Use: Food(x), Person(y), Eats(y, x)]

More fun with sentences

- **"For every food, there is a person who eats that food."**

  [Use: Food(x), Person(y), Eats(y, x)]

- $\forall x\ \exists y\ Food(x) \Rightarrow [\ Person(y) \wedge Eats(y, x)\ ]$

- **Equivalent Forms:**
- $\forall x\ Food(x) \Rightarrow \exists y\ [\ Person(y) \wedge Eats(y, x)\ ]$
- $\forall x\ \exists y\ \neg Food(x) \vee [\ Person(y) \wedge Eats(y, x)\ ]$
- $\forall x\ \exists y\ [\ \neg Food(x) \vee Person(y)\ ] \wedge [\neg\ Food(x) \vee Eats(y, x)\ ]$
- $\forall x\ \exists y\ [\ Food(x) \Rightarrow Person(y)\ ] \wedge [\ Food(x) \Rightarrow Eats(y, x)\ ]$

- **Common Mistakes:**
- $\forall x\ \exists y\ [\ Food(x) \wedge Person(y)\ ] \Rightarrow Eats(y, x)$
- $\forall x\ \exists y\ Food(x) \wedge Person(y) \wedge Eats(y, x)$

More fun with sentences

- **“Every person eats every food.”**

  [Use: Person (x), Food (y), Eats(x, y)]

More fun with sentences

- **"Every person eats every food."**
  [Use: Person (x), Food (y), Eats(x, y)]

- $\forall x\ \forall y\ [\ \text{Person}(x) \wedge \text{Food}(y)\ ] \Rightarrow \text{Eats}(x, y)$

- **Equivalent Forms:**
- $\forall x\ \forall y\ \neg\text{Person}(x) \vee \neg\text{Food}(y) \vee \text{Eats}(x, y)$
- $\forall x\ \forall y\ \text{Person}(x) \Rightarrow [\ \text{Food}(y) \Rightarrow \text{Eats}(x, y)\ ]$
- $\forall x\ \forall y\ \text{Person}(x) \Rightarrow [\ \neg\text{Food}(y) \vee \text{Eats}(x, y)\ ]$
- $\forall x\ \forall y\ \neg\text{Person}(x) \vee [\ \text{Food}(y) \Rightarrow \text{Eats}(x, y)\ ]$

- **Common Mistakes:**
- $\forall x\ \forall y\ \text{Person}(x) \Rightarrow [\text{Food}(y) \wedge \text{Eats}(x, y)\ ]$
- $\forall x\ \forall y\ \text{Person}(x) \wedge \text{Food}(y) \wedge \text{Eats}(x, y)$

More fun with sentences

- **"All greedy kings are evil."**

  [Use: King(x), Greedy(x), Evil(x)]

More fun with sentences

- **“All greedy kings are evil.”**
  [Use: King(x), Greedy(x), Evil(x)]

- $\forall x$ [Greedy(x) $\wedge$ King(x)] $\Rightarrow$ Evil(x)

- **Equivalent Forms:**
- $\forall x$ ¬Greedy(x) $\vee$ ¬King(x) $\vee$ Evil(x)
- $\forall x$ Greedy(x) $\Rightarrow$ [King(x) $\Rightarrow$ Evil(x)]

- **Common Mistakes:**
- $\forall x$ Greedy(x) $\wedge$ King(x) $\wedge$ Evil(x)

More fun with sentences

- **“Everyone has a favorite food.”**

  [Use: Person(x), Food(y), Favorite(y, x)]

More fun with sentences

- **“Everyone has a favorite food.”**

  [Use: Person(x), Food(y), Favorite(y, x)]

- **Equivalent Forms:**
- $\forall x\ \exists y\ \text{Person}(x) \Rightarrow [\ \text{Food}(y) \wedge \text{Favorite}(y, x)\ ]$
- $\forall x\ \text{Person}(x) \Rightarrow \exists y\ [\ \text{Food}(y) \wedge \text{Favorite}(y, x)\ ]$
- $\forall x\ \exists y\ \neg\text{Person}(x) \vee [\ \text{Food}(y) \wedge \text{Favorite}(y, x)\ ]$
- $\forall x\ \exists y\ [\ \neg\text{Person}(x) \vee \text{Food}(y)\ ] \wedge [\ \neg\text{Person}(x) \vee \text{Favorite}(y, x)\ ]$
- $\forall x\ \exists y\ [\text{Person}(x) \Rightarrow \text{Food}(y)\ ] \wedge [\ \text{Person}(x) \Rightarrow \text{Favorite}(y, x)\ ]$

- **Common Mistakes:**
- $\forall x\ \exists y\ [\ \text{Person}(x) \wedge \text{Food}(y)\ ] \Rightarrow \text{Favorite}(y, x)$
- $\forall x\ \exists y\ \text{Person}(x) \wedge \text{Food}(y) \wedge \text{Favorite}(y, x)$

More fun with sentences

- **“There is someone at UCI who is smart.”**

  [Use: Person(x), At(x, UCI), Smart(x)]

More fun with sentences

- **"There is someone at UCI who is smart."**

  [Use: Person(x), At(x, UCI), Smart(x)]

- $\exists x \; \text{Person}(x) \wedge \text{At}(x, \text{UCI}) \wedge \text{Smart}(x)$

- **Common Mistakes:**
- $\exists x \; [\, \text{Person}(x) \wedge \text{At}(x, \text{UCI}) \,] \Rightarrow \text{Smart}(x)$

More fun with sentences

- **"Everyone at UCI is smart."**

  [Use: Person(x), At(x, UCI), Smart(x)]

More fun with sentences

- **"Everyone at UCI is smart."**
  [Use: Person(x), At(x, UCI), Smart(x)]
- $\forall x$ [Person(x) $\wedge$ At(x, UCI)] $\Rightarrow$ Smart(x)
- **Equivalent Forms:**
- $\forall x$ $\neg$[Person(x) $\wedge$ At(x, UCI)] $\vee$ Smart(x)
- $\forall x$ $\neg$Person(x) $\vee$ $\neg$At(x, UCI) $\vee$ Smart(x)
- **Common Mistakes:**
- $\forall x$ Person(x) $\wedge$ At(x, UCI) $\wedge$ Smart(x)
- $\forall x$ Person(x) $\Rightarrow$ [At(x, UCI) $\wedge$ Smart(x)]
-

More fun with sentences

- **“Every person eats some food.”**

  [Use: Person (x), Food (y), Eats(x, y)]

More fun with sentences

- **"Every person eats some food."**

  [Use: Person (x), Food (y), Eats(x, y)]

- $\forall x \, \exists y \; Person(x) \Rightarrow [\, Food(y) \wedge Eats(x, y) \,]$
-
- **Equivalent Forms:**
- $\forall x \; Person(x) \Rightarrow \exists y \, [\, Food(y) \wedge Eats(x, y) \,]$
- $\forall x \, \exists y \; \neg Person(x) \vee [\, Food(y) \wedge Eats(x, y) \,]$
- $\forall x \, \exists y \; [\, \neg Person(x) \vee Food(y) \,] \wedge [\, \neg Person(x) \vee Eats(x, y) \,]$

- **Common Mistakes:**
- $\forall x \, \exists y \; [\, Person(x) \wedge Food(y) \,] \Rightarrow Eats(x, y)$
- $\forall x \, \exists y \; Person(x) \wedge Food(y) \wedge Eats(x, y)$
-

More fun with sentences

- **“Some person eats some food.”**

  [Use: Person (x), Food (y), Eats(x, y)]

More fun with sentences

- **“Some person eats some food.”**

  [Use: Person (x), Food (y), Eats(x, y)]

- $\exists x \ \exists y \ Person(x) \land Food(y) \land Eats(x, y)$

- **Common Mistakes:**

- $\exists x \ \exists y \ [\ Person(x) \land Food(y)\ ] \Rightarrow Eats(x, y)$

Summary

- First-order logic:
  - Much more expressive than propositional logic
  - Allows objects and relations as semantic primitives
  - Universal and existential quantifiers
- Syntax: constants, functions, predicates, equality, quantifiers
- Nested quantifiers
  - Order of unlike quantifiers matters (the outer scopes the inner)
    - Like nested ANDs and ORs
  - Order of like quantifiers does not matter
    - like nested ANDS and ANDs
- Translate simple English sentences to FOPC and back