

1. (20 points total, 5 pts each) Which of the following are true and which are false? Explain your answers. Unless stated otherwise, assume a finite branching factor, step costs $\geq \epsilon > 0$, and at least one goal at a finite depth. You may be in either a tree or a graph.

a. (5 pts) Depth-first search always expands at least as many nodes as A* search with an admissible heuristic.

FALSE. Depth-first search may possibly, sometimes, BY GOOD LUCK, expand fewer nodes than A* search with an admissible heuristic. E.g., it is logically possible that sometimes, by good luck, depth-first search may march directly to the goal with no back-tracking.

b. (5 pts) $h(n)=0$ is an admissible heuristic for the 8-puzzle.

TRUE. $h(n)=0$ NEVER over-estimates the remaining optimal distance to a goal node.

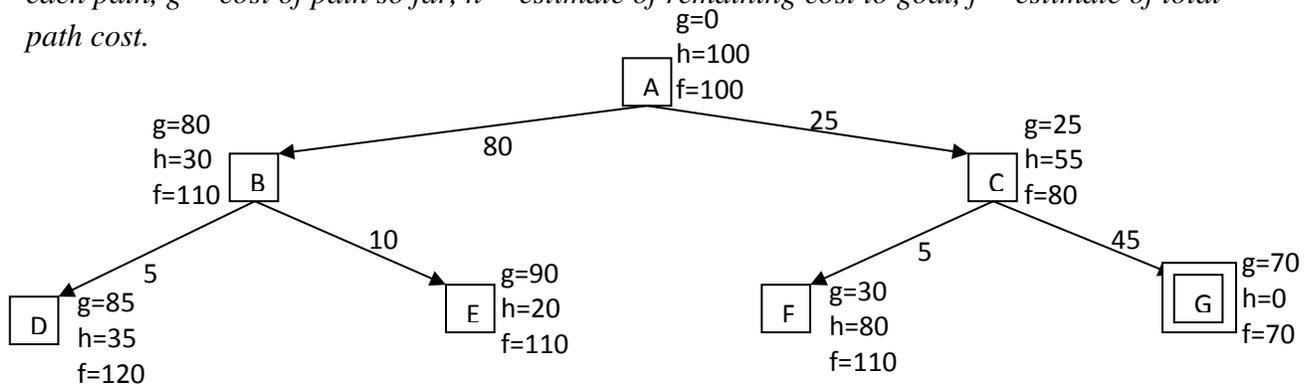
c. (5 pts) Breadth-first search is complete whenever the branching factor is finite, even if zero step costs are allowed.

TRUE, because if there exists a goal it occurs at finite depth d and will be found in $O(b^d)$ steps. “Complete” means “will find a goal when one exists” --- and does NOT imply “optimal,” which means “will find a lowest-cost goal when one exists.” Thus, the step costs are irrelevant to “complete.”

d. (5 pts) Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces; then Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

FALSE. The Manhattan distance may over-estimate the optimal remaining number of moves to the goal because a rook may cover several squares in a single move. NOTE: If the path cost instead were the number of squares covered, then Manhattan distance would be admissible.

2. (20 points total, 5 pts off for each wrong answer, but not negative) Use the following tree to indicate the order that nodes are expanded, for different types of search. Assume that A is the start node and G (double box) is the only goal node. Here, path costs are shown to the right of each path, g = cost of path so far, h = estimate of remaining cost to goal, f = estimate of total path cost.



a. (5 pts) *Uniform-cost search.*

A, C, F, (G).

b. (5 pts) *Iterative deepening depth-first search.*

A; A, B, C, (G).

c. (5 pts) *Greedy best-first search.*

A, B, E, D, C, (G).

d. (5 pts) *A* search.*

A, C, (G).

e. (5 pts) *Is the heuristic h admissible?*

NO. For example, $h(A)=100$, but the optimal cost to the goal node G is only 70. Thus, $h(A)$ sometimes OVER-ESTIMATES the remaining optimal distance to G, and so h is not admissible.

f. (5 pts) *Relabel the heuristic values h so that h is admissible but not consistent.*

There are many possibilities. One minimal change is to set $h(A)=60$ and $h(C)=20$. This is admissible because $h(A) = 60 \leq h^*(A) = 70$ and $h(C) = 20 \leq h^*(C) = 45$ (recall that h^* is the true optimal remaining distance to the goal). It is not consistent because $f(n)$ decreases along the path A-C-G, i.e., $f(A) = g(A)+h(A) = 0+60 = 60 > 45 = 25+20 = \text{cost}(A,C)+h(C) = g(C)+h(C) = f(C)$.

Any solution satisfying the following criteria should receive full credit:

Admissible:

$$70 \geq h(A)$$

$$45 \geq h(C)$$

Not consistent:

$$h(A) > \text{cost}(A,C)+h(C) = 25+h(C)$$

Admissible and not consistent:

$$70 \geq h(A) > \text{cost}(A,C)+h(C) = 25+h(C)$$

3. (20 points total, 5 pts off for each wrong answer, but not negative) Recall that

* True path cost so far = $g(n)$.

* Estimated cost to goal = $h(n)$.

* Estimated total cost = $f(n) = g(n) + h(n)$:

The following is a proof that A* tree search (queue sorted by f) is optimal if the heuristic h is admissible. The lines have been labelled A through G. Unfortunately, they have been scrambled.

Let ng be the first goal node popped off the queue. Let no be any other node on the queue. We wish to prove that no can never be extended to a path to any goal node that costs less than the path to ng that we just found.

A : true total cost of ng

F: = $g(ng)$ // because ng represents a complete path

D: = $f(ng)$ // by definition of f with $h(ng) = 0$ since ng is a goal

B : $\leq f(no)$ // because queue is sorted by f

E: = $g(no) + h(no)$ // by definition of f

C : $\leq g(no) +$ true cost to goal from no // because h is admissible

G : = true total cost of no extended to a path to any goal node

Fill in the blanks with the letters B, C, D, E, F, and G in the correct order to prove that the true total cost of $ng \leq$ true total cost of no . The first and last letters, A and G, have been done for you as an example.

A F D B E C G