Quadcopter Fluid Mechanics

October 22, 2018

Overview

- What generates a lift? Discussion of Bernoulli's Equation
- Parameters of the rotor
- Thrust Equation
- Hints for HW2

Introduction

- Air flow is generated by a motor spinning with a propeller
- Thrust is caused by the path of air flow over the propeller
- Electric motor spins faster to generate more air flow and thrust

Introduction

- Propellers are shaped differently for counterclockwise and clockwise rotation.
- Most hobby propellers come with two or three blades per propeller.
-

ENGR 7A: 2-blade propellers

The Thrust Equation

$$T = 2\left(\frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta} - 1\right]\right)^2 \rho(\Omega R)^2 A$$

a: two dimensional lift slope factor usually has a value of 5.7

R: propeller radius

c: propeller chord

Ω: propeller angular velocity

Θ: propeller pitch

Rotor solidity:

$$s = \frac{Propeller\ Area}{Total\ Disc\ Area}$$

$$= \frac{NcR}{\pi R^2} = \frac{Nc}{\pi R}$$

N: number of blades per propeller

Bernoulli's Equation

Assume steady incompressible flow:

• **Steady** means that if we sit at a fixed point in space, things don't change with time.

• **Incompressible** flow means that we can approximate the density to be constant, ρ = *density* = constant

Under these conditions, the Bernoulli's Equation is

$$p + \frac{1}{2}\rho V^2 = p_0$$

Static pressure

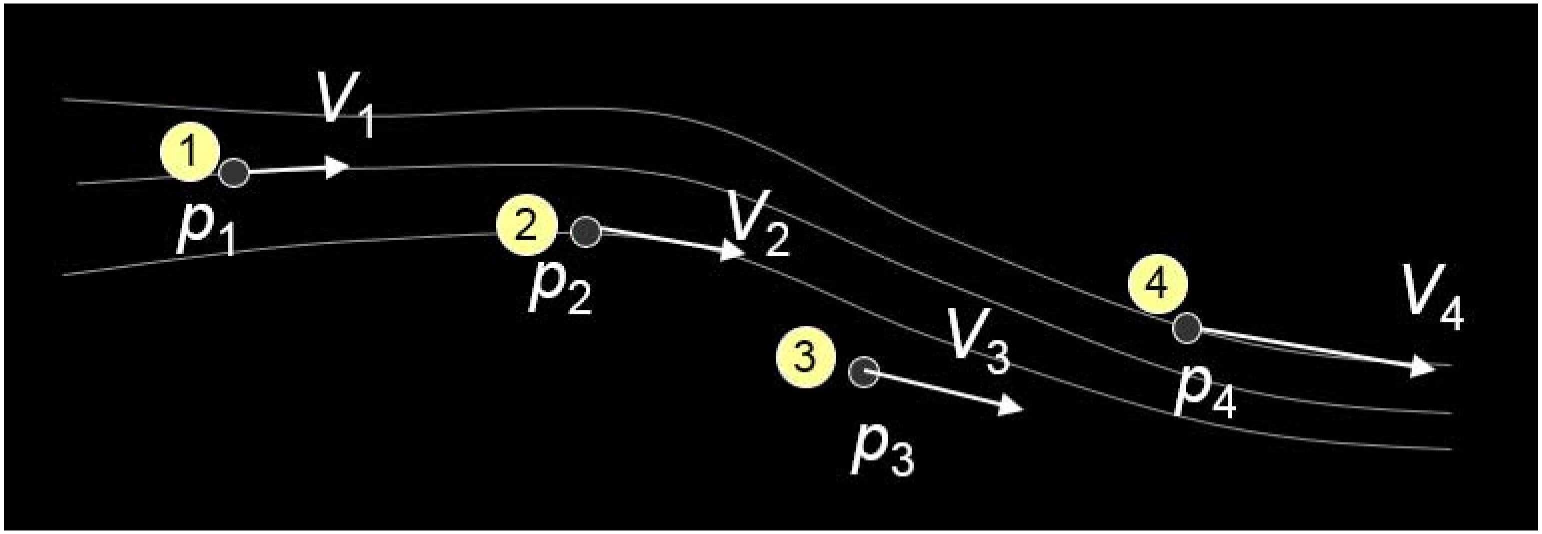
Dynamic pressure

Total pressure
Constant everywhere

Bernoulli's Equation

Under the condition of **steady incompressible** flow, if we know the total pressure p_o is constant at all points, then we can relate the static pressure and velocity at all points:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 = p_3 + \frac{1}{2}\rho V_3^2 = \cdots = p_0$$



Bernoulli's Equation

Given that the total pressure is constant, what happens to the static pressure when the velocity increases?

$$p + \frac{1}{2}\rho V^2 = p_0$$

Static pressure

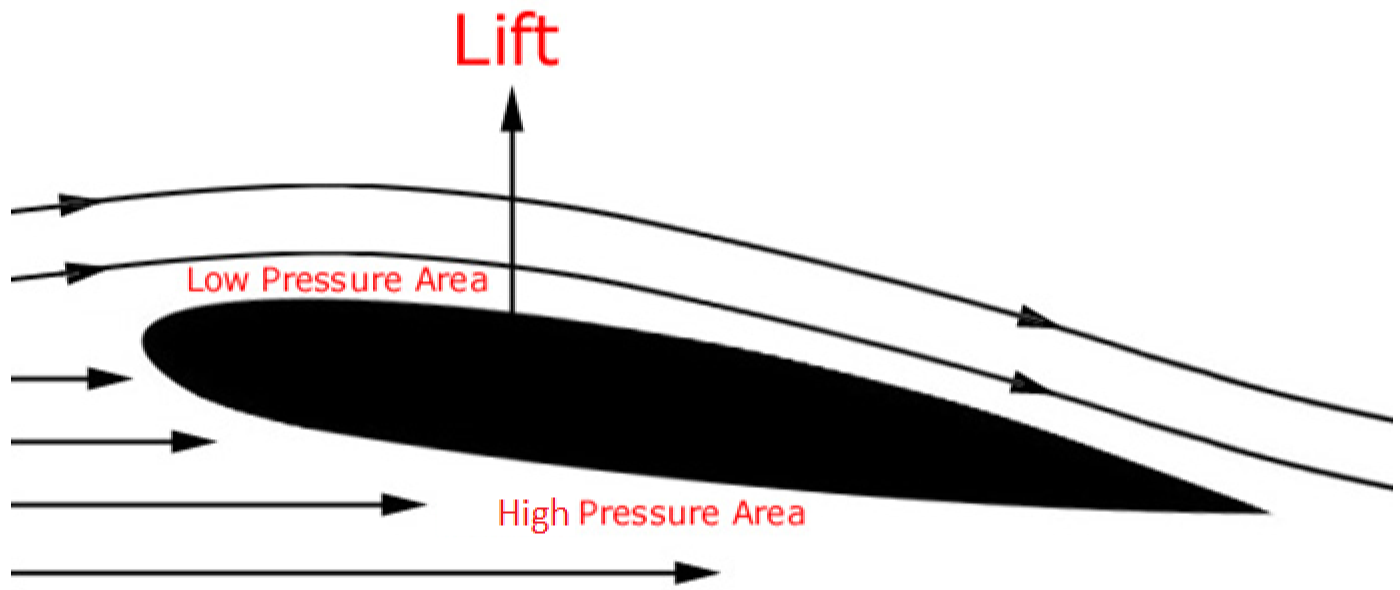
Dynamic pressure

Total pressure
Constant everywhere

- https://www.youtube.com/watch?v=7nI2RwjeZWk

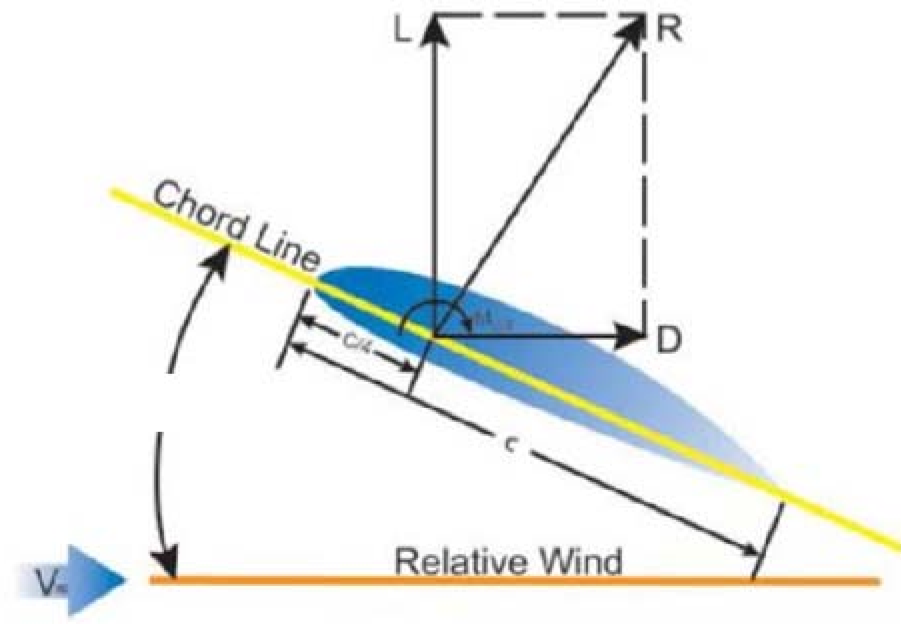
Next: Apply this concept the an airfoil to generate lift

Causes of Lift



- The curve and angle of the airfoil makes the air velocity faster above the airfoil and slower below the airfoil
- Faster velocity produces lower pressure above airfoil
- The airfoil gets “sucked up” by pressure difference

Causes of Lift

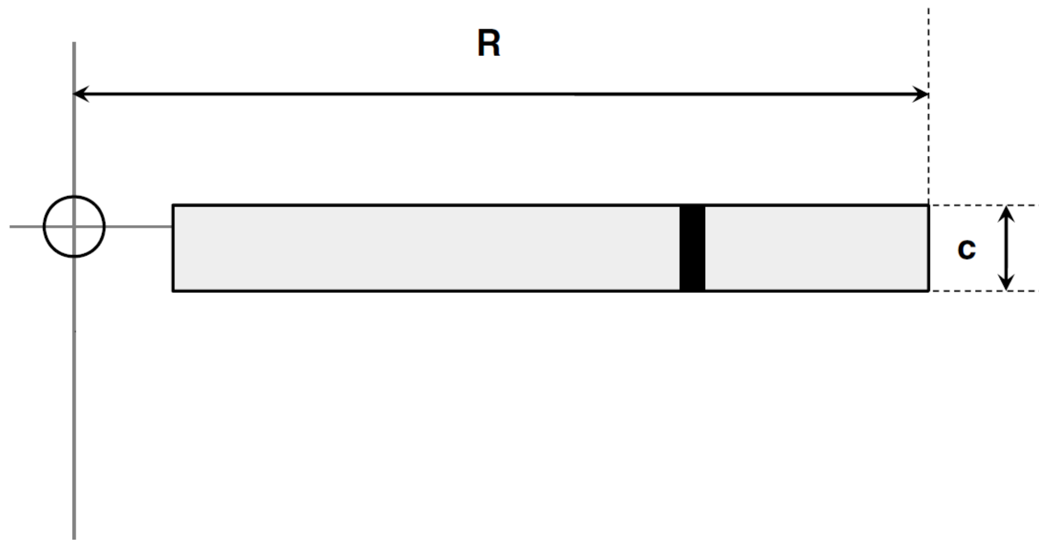


An airfoil only generates one total force (R)

- The component of R in the vertical direction is called lift (L)
- The component of R in the horizontal direction is called drag (D)
- The goal is to maximize lift and minimize drag by changing propeller pitch (θ)

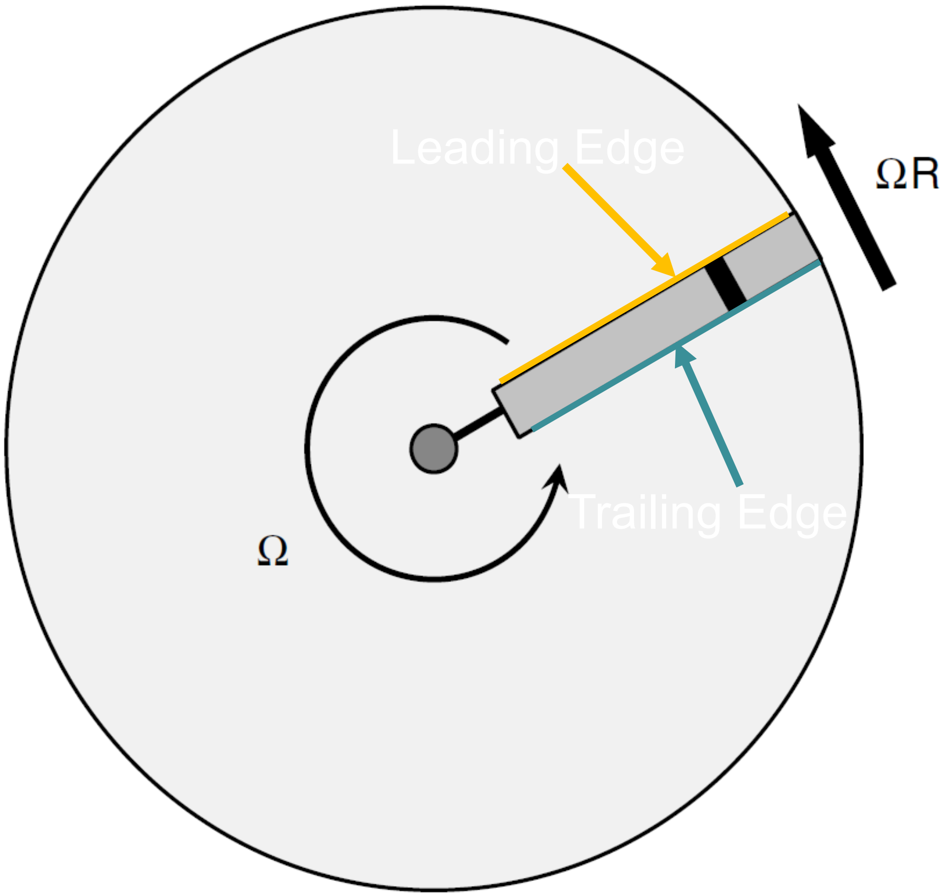
http://www.youtube.com/watch?v=xW63SZ1LAqo

Propeller Parameters



R: propeller radius
c: propeller chord

Propeller Parameters



Ω: propeller angular velocity

Tip velocity $V_T = \Omega R$

Rotor solidity:

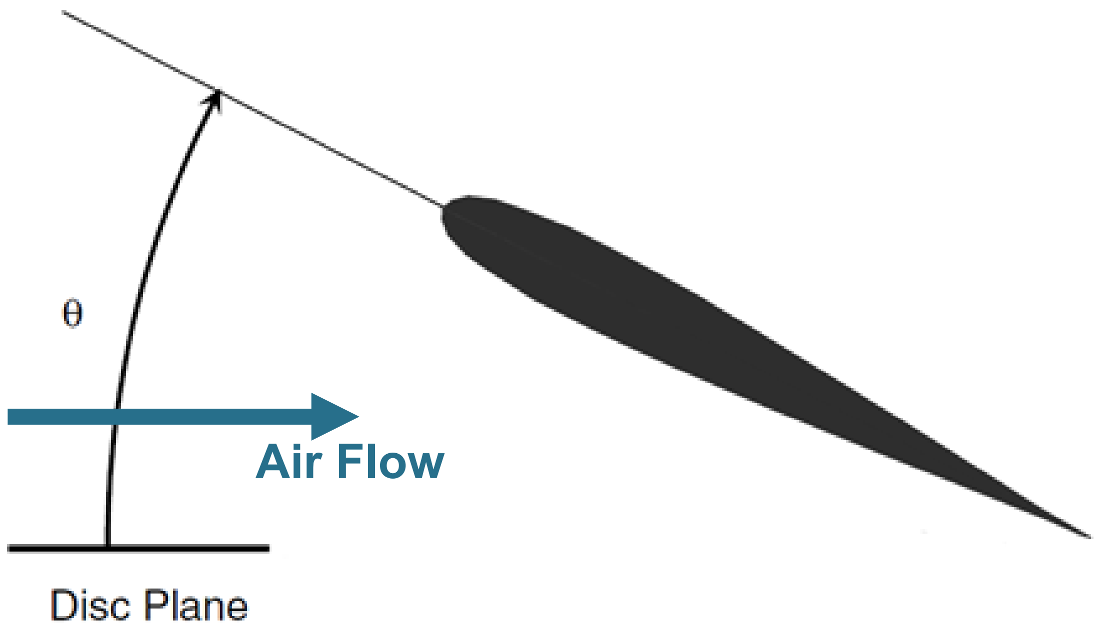
$$s = \frac{Propeller\ Area}{Total\ Disc\ Area}$$

$$= \frac{NcR}{\pi R^2} = \frac{Nc}{\pi R}$$

N: number of blades per propeller

Propeller Parameters

θ: Propeller pitch



What direction is the force if the leading edge is LOWER than the trailing edge?

Quadcopter Fluid Mechanics

Bernoulli's Equation: $p + \frac{1}{2}\rho V^2 = p_0$

Pressure = Force x Area $p = \frac{F}{A}$

Propeller Parameters:

R = propeller radius

A = propeller disc area = πR^2

Ω = angular velocity

$V_T = \Omega R$ = tip velocity

Derivation of Thrust Equation

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Use fluid mechanics and aerodynamic theory to derive an equation for the thrust of a propeller.

Bernoulli's Equation:

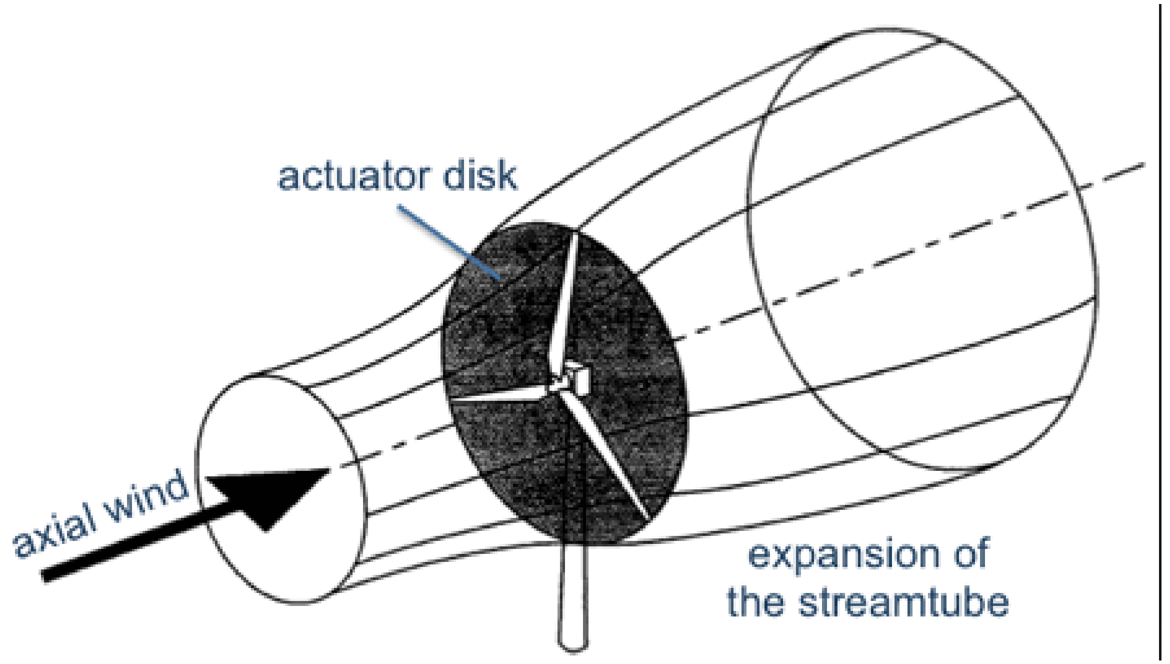
$$p + \frac{1}{2}\rho V^2 = p_0$$

+ Disc Actuator Theory & Blade Element Theory

NOTE: You do not need to memorize the derivation or equations. This is a demonstration of how equations are combined to produce a useful equation.

Disc Actuator Theory

Approximate the quadcopter propeller as a disc



Disc Actuator Theory

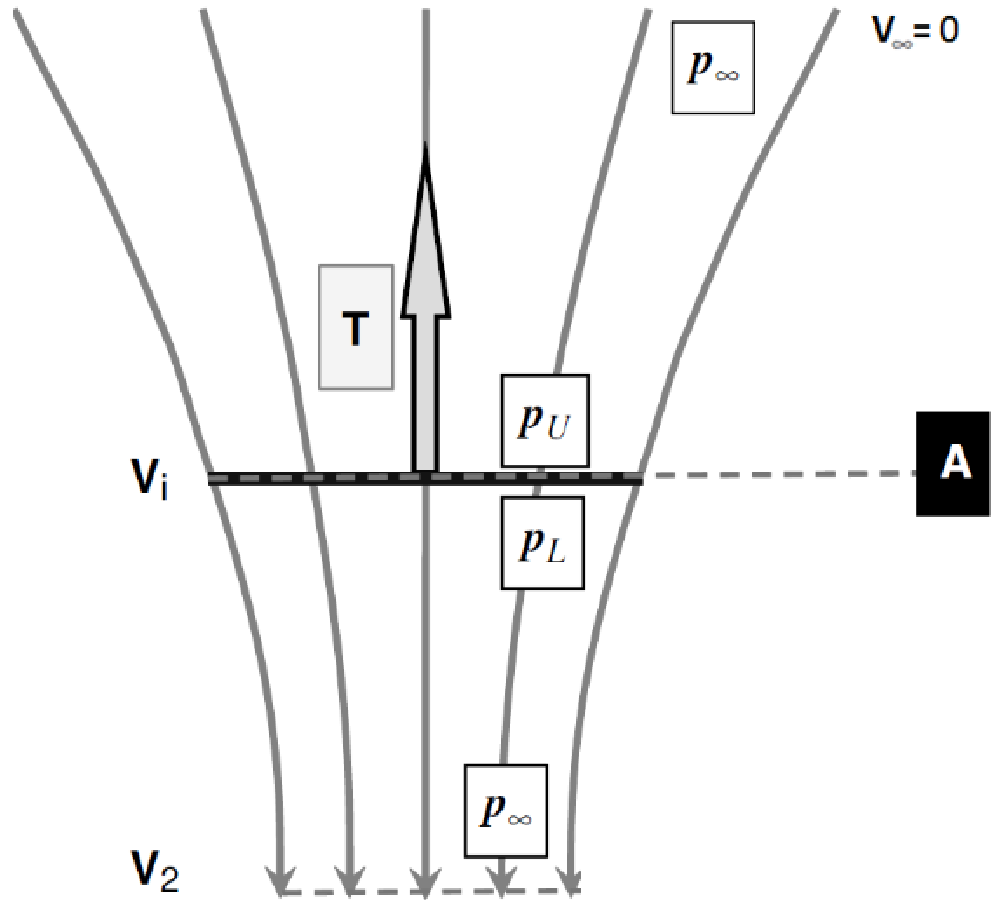
- Air flow is modeled as a tube
- Propeller is modeled as a 1-D cross-section of the tube

T: Thrust

A: Propeller disc area

V_i: Induced velocity

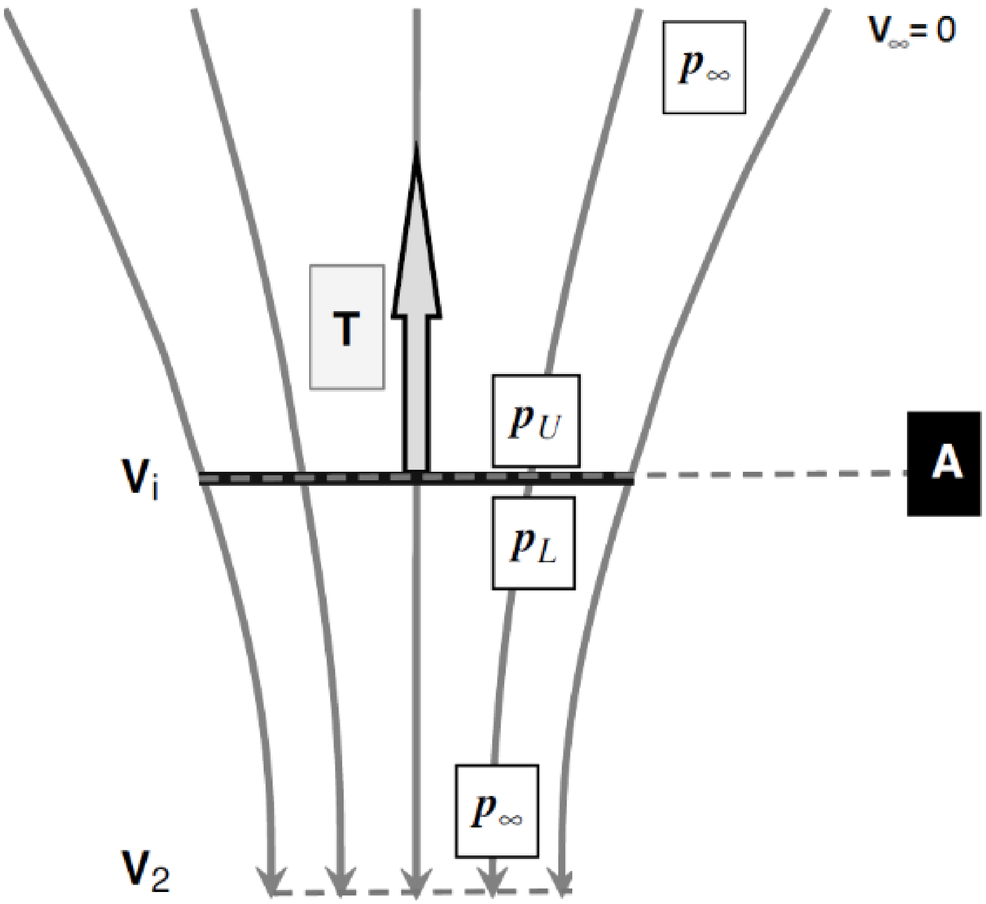
V_2: Downstream velocity



http://www.youtube.com/watch?v=rxO5DdkJhcc

Disc Actuator Theory

- p_U: Pressure above the prop
- p_L: Pressure below the prop
- p_∞: Pressure away from prop



Bernoulli's Equation Applied to Quadcopter

Bernoulli's equation above the propeller:

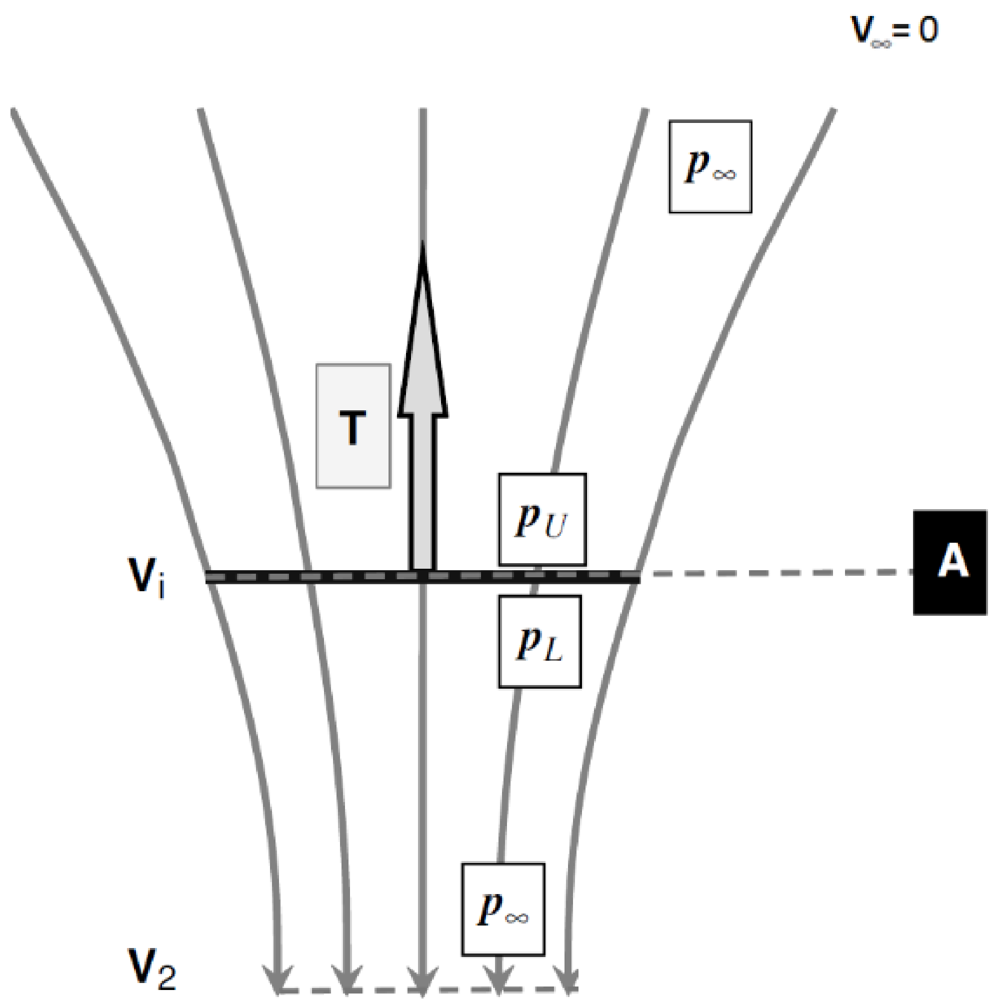
$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p_U + \frac{1}{2}\rho V_i^2$$

Bernoulli's equation below the propeller:

$$p_\infty + \frac{1}{2}\rho V_2^2 = p_L + \frac{1}{2}\rho V_i^2$$

→ Subtracting equations:

$$p_L - p_U = \frac{1}{2}\rho (V_2)^2$$



Definition of Thrust

Thrust is a **FORCE**

Representations of Force:

1. Change of momentum with respect to time

$$F = \frac{d}{dt}(mV) = m\frac{d}{dt}(V) = ma$$

2. Pressure x Area

$$P = \frac{F}{A}$$

Remember Force Units $= kg * m/s^2$

Definition of Thrust

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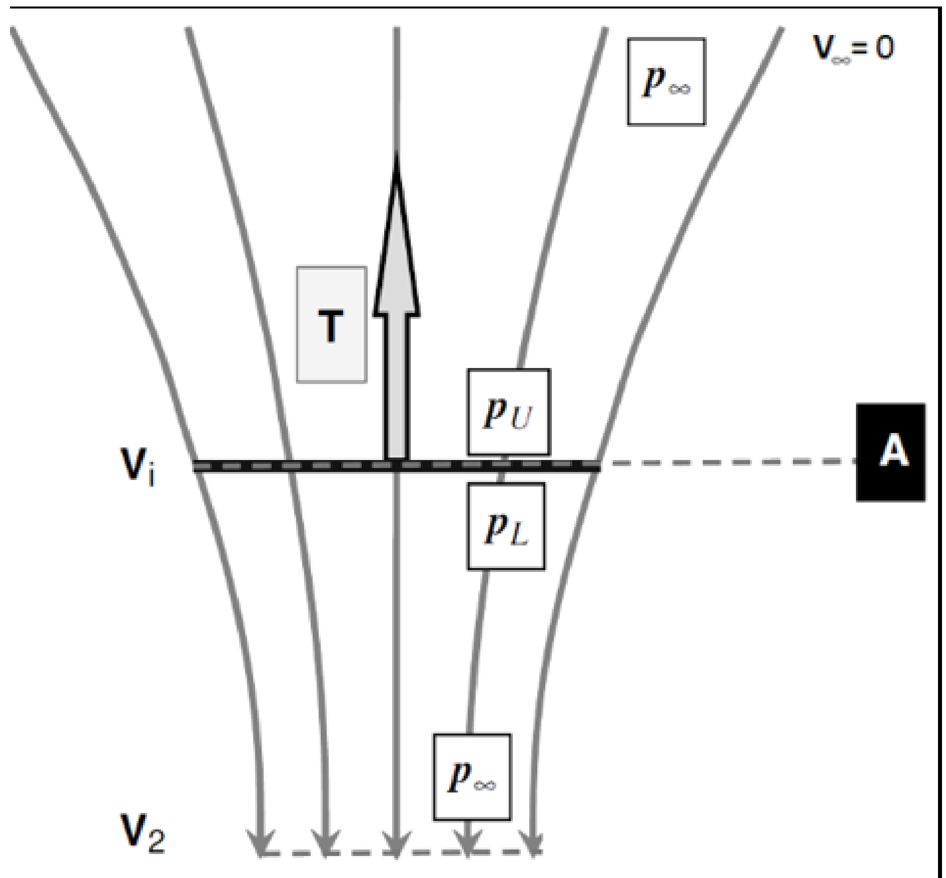
- Rate of change of momentum gives the rotor thrust:

$$T = \rho A V_i V_2$$

- Thrust can also be represented in terms in pressure differences

$$T = A(p_L - p_U)$$



Solving for Induced Velocity

Recall that:

$$T = \rho A V_i V_2$$

$$T = A(p_L - p_U)$$

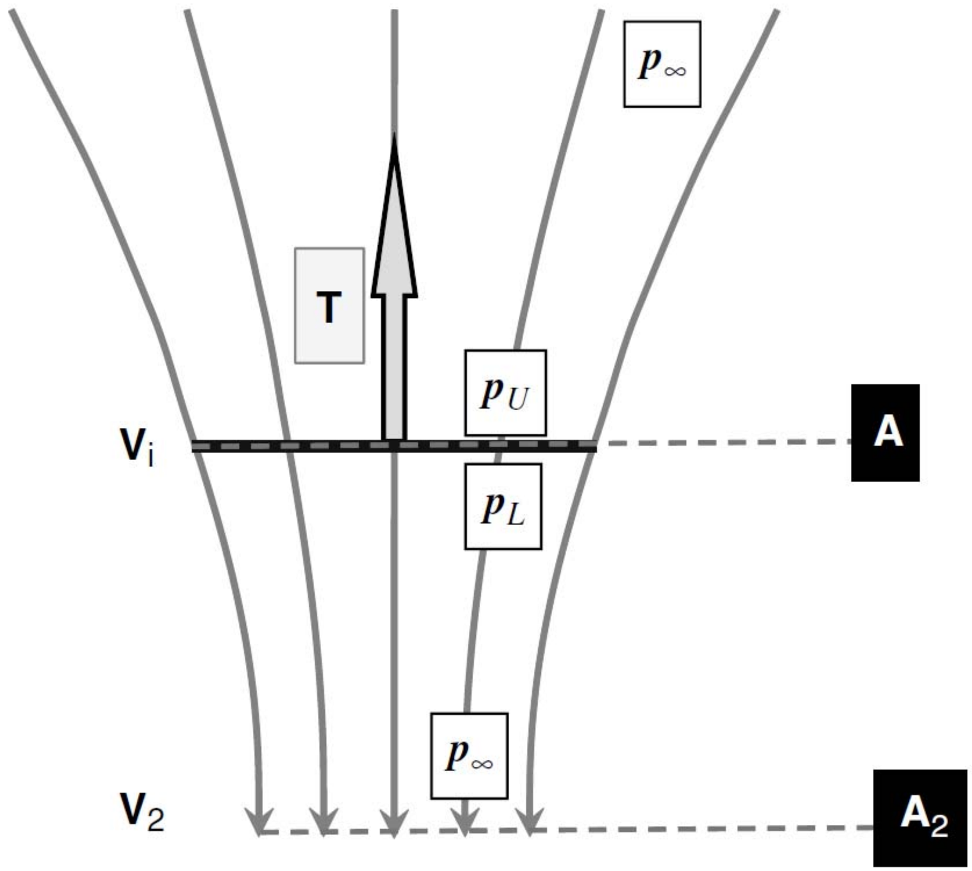
$$p_L - p_U = \frac{1}{2}\rho V_2^2$$

- Assembling above equations gives:

$$V_2 = 2V_i$$

- Plugging into first equation, induced velocity:

$$V_i = \sqrt{\frac{T}{2\rho A}}$$



Thrust Coefficient

- "Non-dimensionalized" induced velocity:

$$\lambda = \frac{V_i}{V_T}$$

where V_T is the tip velocity.

$$V_T = \Omega R$$

- Thrust coefficient defined as:

$$C_T = \frac{T}{\frac{1}{2}\rho V_T^2 A}$$

- Plug in the induced velocity equation into non-dimensionalized equation, solve for V_T^2, and plug into thrust coefficient equation:

$$\lambda = \frac{1}{2}\sqrt{C_T}$$

$$C_T = 4\lambda^2$$

Blade Element Theory

Definition from blade element theory, non-dimensionalized velocity λ:

$$\lambda = \frac{sa}{16}\left[\sqrt{1 + \frac{64}{3sa}\theta} - 1\right]$$

Recall:

$$C_T = 4\lambda^2$$

$$T = C_T * \left(\frac{1}{2}\rho V_T^2 A\right)$$

Resulting in:

$$T = 2\left(\frac{sa}{16}\left[\sqrt{1 + \frac{64}{3sa}\theta} - 1\right]\right)^2 \rho(\Omega R)^2 A$$

a: two dimensional lift slope factor usually has a value of 5.7

Rotor Power Equation

Definition:

$$P_i = T\,V_i$$

Recall:

$$T = 2\left(\frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta}-1\right]\right)^2 \rho(\Omega R)^2 A$$

$$\lambda = \frac{V_i}{V_T}$$

$$\lambda = \frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta}-1\right]$$

Then the equation for power is:

$$P = 2\left(\frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta}-1\right]\right)^3 \rho(\Omega R)^3 A$$

Design Parameters

$$T = 2\left(\frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta} - 1\right]\right)^2 \rho(\Omega R)^2 A$$

$$P = 2\left(\frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta} - 1\right]\right)^3 \rho(\Omega R)^3 A$$

- Increase in propeller pitch, radius, and angular velocity result in an increase in thrust and power.
  - Increasing Power will decrease the Battery Life.

Props and Motors

<u>ENGR 7A Options:</u>
5” or 6” or 8” propellers
Motors with maximum RPM’s ranging from 13320 RPM’s to 25530 RPM’s

Example Problem

Given:

$$R = 2.5in, \qquad c = 0.58in, \qquad N = 2, \qquad \rho = 1.1839\frac{kg}{m^3}$$

$$\Omega = 10{,}000\frac{rev}{min}, \qquad \theta = 20°,$$

Convert to metric units:

$$R = 0.0635m, \qquad c = 0.0147m,$$

$$\Omega = 10{,}000\frac{rev}{min} \times \frac{2\pi}{1\,rev} \times \frac{1min}{60s} = 1047.1976\frac{rad}{s},$$

$$\theta = 20° \times \frac{2\pi\,rad}{360°} = 0.3491rad$$

Example Problem

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$$s = \frac{2(0.0147m)}{\pi(0.0635m)} = 0.1477,$$

$$A = \pi R^2 = \pi(0.0635)^2 = 0.0127m^2$$

$$T = 2\left(\frac{sa}{16}\left[\sqrt{1 + \frac{64}{3sa}\theta} - 1\right]\right)^2 \rho(\Omega R)^2 A$$

$$T = 2\left(\frac{(0.1477)(5.7)}{16}\left[\sqrt{1 + \frac{64}{3(0.1477)(5.7)}0.3491rad} - 1\right]\right)^2 1.1839\frac{kg}{m^3}\left(\left(1047.1976\frac{\text{rad}}{\text{s}}\right)(0.0635m)\right)^2 (0.0127m^2) = 1.6781N$$

Example Problem

$$T = 1.6781N$$

$$1.6781\ kg\frac{m}{s^2} \times \frac{1}{9.8\frac{m}{s^2}} \times \frac{1000g}{1kg} = 171.231g$$

Multiply by 4 to get total quadcopter thrust.

Thrust : Weight Ratio

2 : 1 → 50% Throttle required to lift off

3 : 1 → 33% Throttle required to lift off

Design Requirements

Performance: Fly the quadcopter for at least 5 minutes.

In Lab Experiment Week 4: Thrust vs. Current Draw

Procedure: Run motor with propeller at max throttle and several other thrusts and record thrust produced and current draw.

When thrusts and corresponding current levels are known, you can calculate the current required to hover (thrust = weight), and then estimate flight time for given battery capacity.

Selecting Motors

Parameters to consider for motor selection:

- Thrust and power = performance
- Current draw = endurance
- Size = layout

Selecting Motors

See Purchase Order (PO) Form Instructions & Student Parts List on Course Website.

Remember: Not all motors support all propellers

Faster motors use only smaller propellers

Recommended Schedule

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Concept Design and Preliminary Dimensions – end of week 4

Motor and propeller selection – end of week 4

Completed Solidworks Model and PO Form – end of week 5 – GRADED DEADLINE

Completed Fabrication of structure – end of week 7 – GRADED DEADLINE

Completed Wiring electrical components – weeks 8-9

Quadcopter Testing – weeks 8-10

Final quadcopter testing/ qualifying – week 10

Final Competition

Hints for HW2

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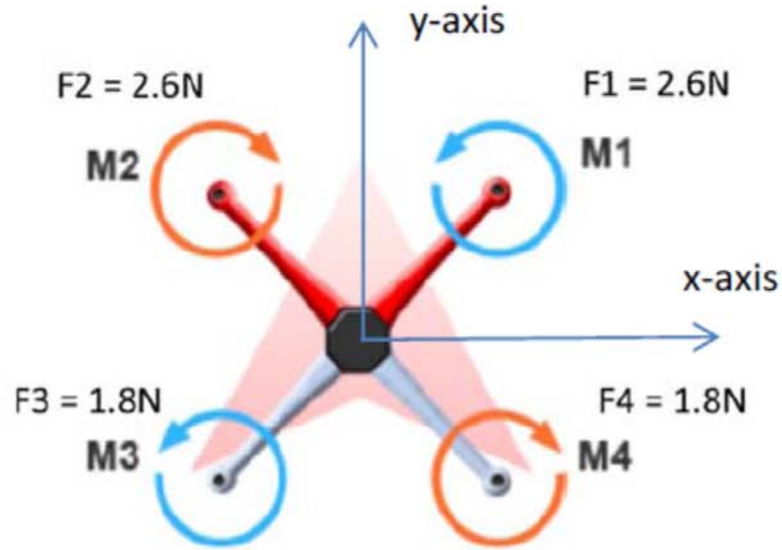
1. See the quadcopter configuration to the right. The forces generated by four motors (M1, M2, M3, M4) are listed in the picture with the red arrow indicating the front of the quadcopter. The motor to motor distance is 12 inches. The x and y axes are labeled in the positive direction, and the z-axis is positive going out of the plane of the paper.



   a. Identify which axis the quadcopter will rotate around.
   b. Determine whether this motion is considered roll, pitch, or yaw.
   c. Determine the moment/torque generated. Is the generated moment in the positive or negative direction?

Reading Assignment for Week 4

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"Introduction to Engineering Design" Book 11

Engineering Skills and Quadcopter Missions 4th Edition 2017

Chapter 4 "Fluid Mechanics and Design Analysis"

Announcement: Don't forget to read "Electrical Safety" pdf file (Canvas Week 3) to prepare for lab safety quiz this week.