Quadcopter Stability and Circuit Analysis

November 5, 2018

Overview

- Quadcopter Stability
  - Moment of Inertia
  - Scaling
  - Ground Effect

- Electric Circuit Analysis

Moment of Inertia

- An important value that effects the movement of a quadcopter is the moment of inertia.
  - Moment of inertia gives an indication of an object's resistance to a torque about a rotation axis.
- Derived from Newton's second law of motion and the sum of moments:

$$\sum \boldsymbol{M} = \frac{d}{dt}(I\omega)$$

where I is the moment of inertia and ω is the angular velocity.

Moment of Inertia

- From the previous slide, the sum of the moments is:

$$\sum \boldsymbol{M} = \frac{d}{dt}(I\omega)$$

- Since the moment of inertia is a scalar and the angular velocity is a vector, the sum of the moments can be written as:

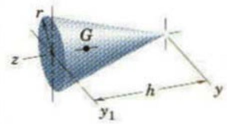
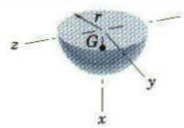
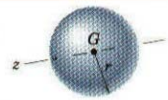
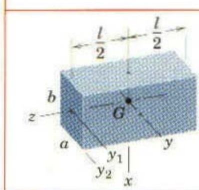
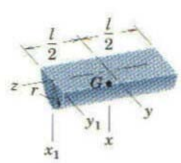
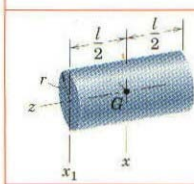
$$\sum \boldsymbol{M} = I\frac{d\omega}{dt} = I\alpha.$$

- The definition of the moment of inertia is:

$$I = \int r^2\, dm$$

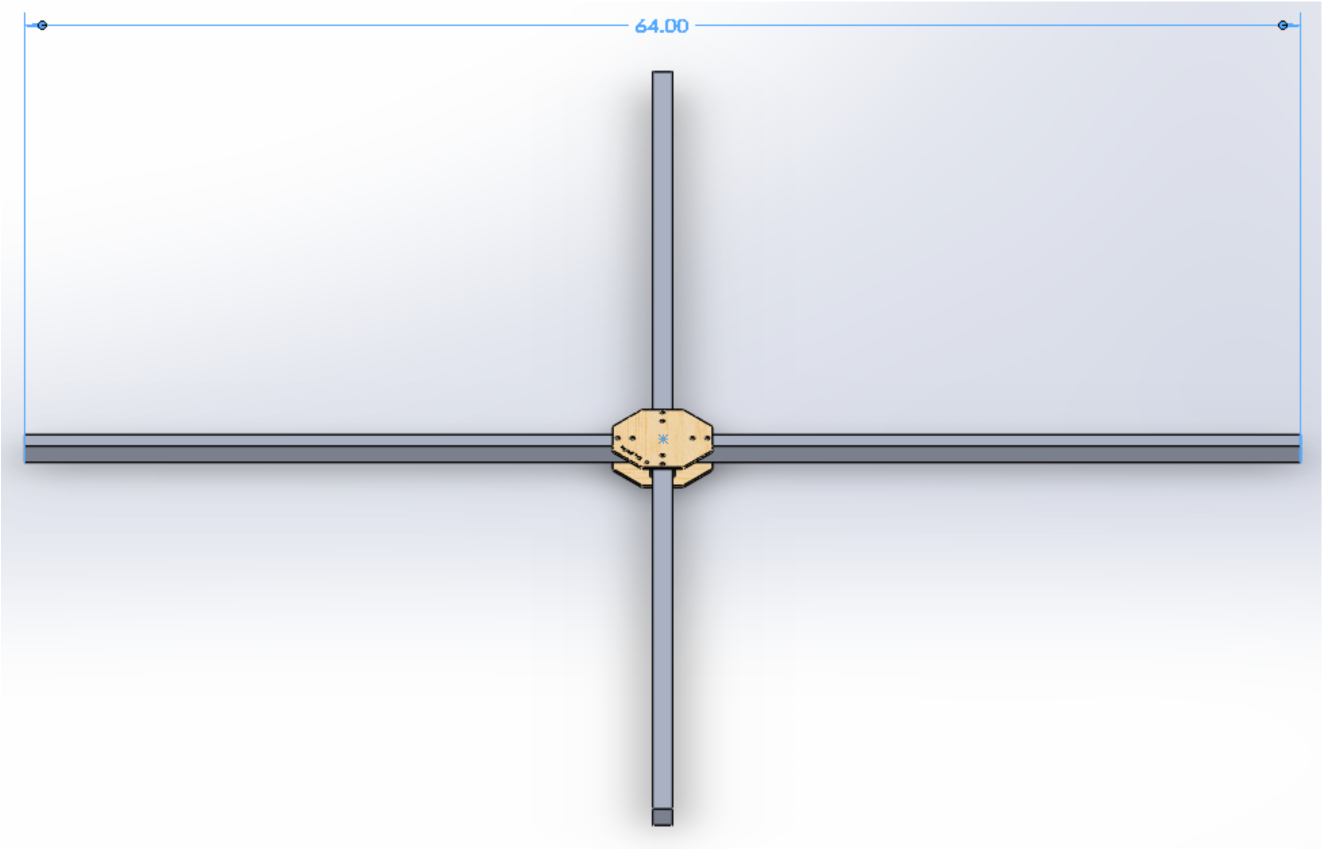
A decrease in the moment of inertia will result in a decrease in the moment required to reach a desired angular acceleration.

Moment of Inertia

	Right-Circular Cone	$\bar{z} = \frac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$
	Hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320}mr^2$
	Sphere	—	$I_{zz} = \frac{2}{5}mr^2$
	Rectangular Parallelepiped	—	$I_{xx} = \frac{1}{12}m(a^2 + l^2)$ $I_{yy} = \frac{1}{12}m(b^2 + l^2)$ $I_{zz} = \frac{1}{12}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_2y_2} = \frac{1}{3}m(b^2 + l^2)$
	Semicylinder	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Circular Cylinder	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$

Scaling

- Scaling the size of the quadcopter alters the mass and moment of inertia.
  - The moment of inertia is affected more dramatically.



Scaling

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Radius (in)	Mass (lbs.)	Ixx (lbs*in^2)	Iyy (lbs*in^2)	Izz (lbs*in^2)
4	1.27	4.91	8.78	4.91
8	2.89	35.84	69.34	35.84
16	6.12	277.02	553	277.02
32	10.98	1731.28	3453.76	1731.28

Scaling

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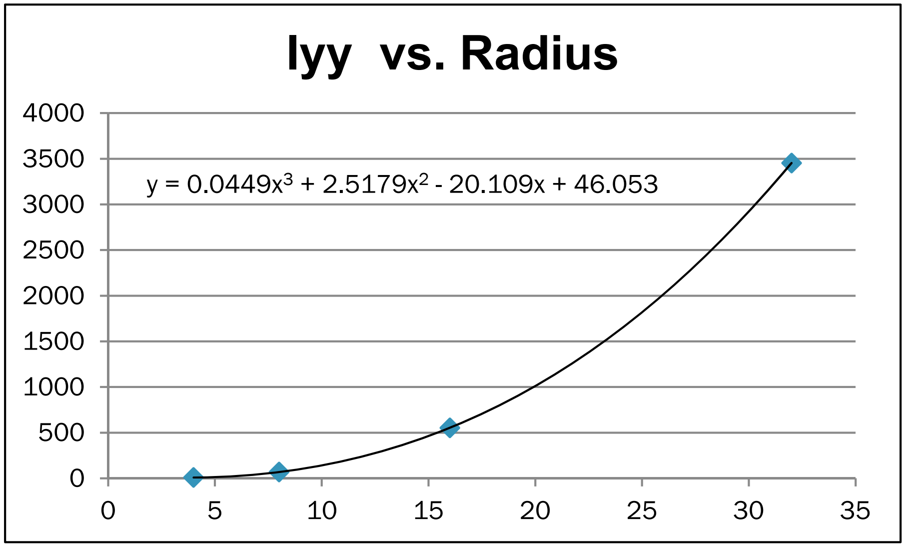
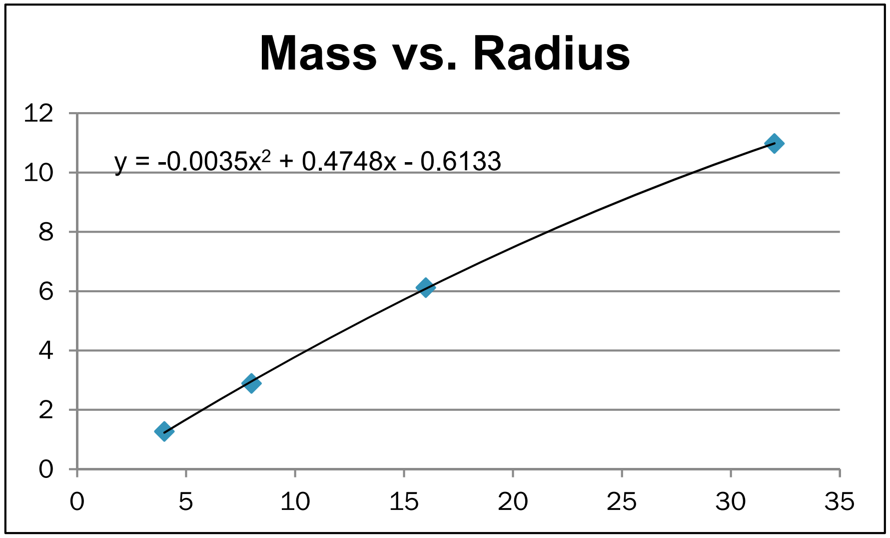
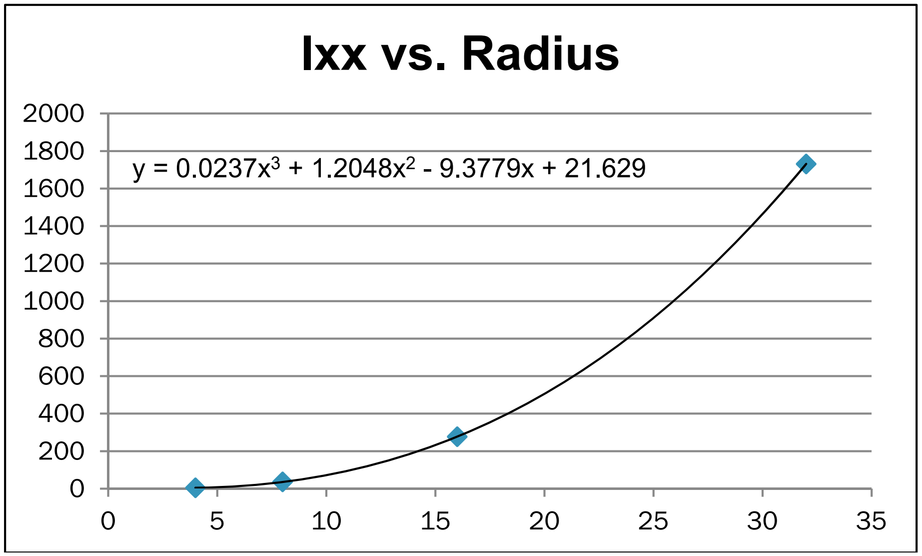
  - The moment of inertia is affected more dramatically.







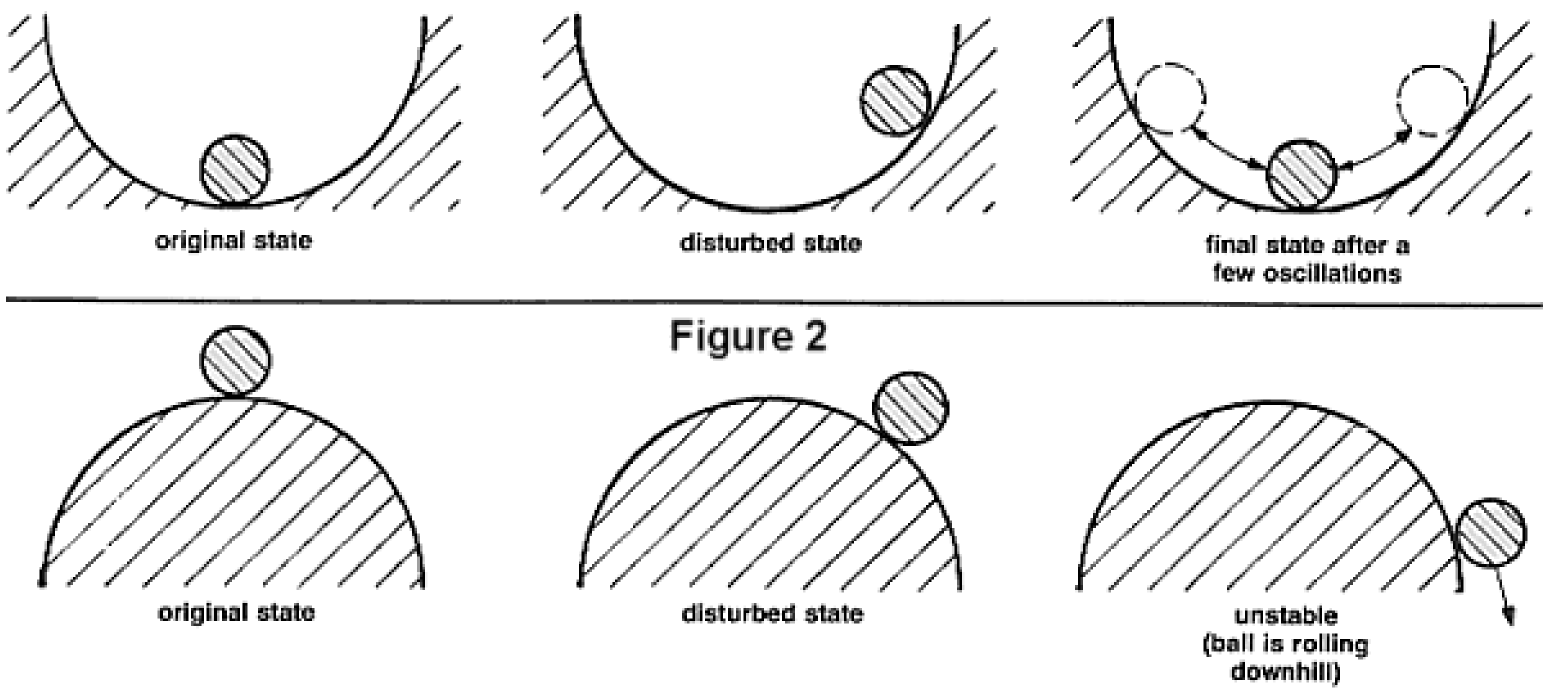
Scaling

- **A larger quadcopter will require larger moments to maneuver**
  - Less likely to be affected by outside perturbations.

- **A smaller quadcopter will require smaller moments to maneuver**
  - More likely to be affected by outside perturbations.

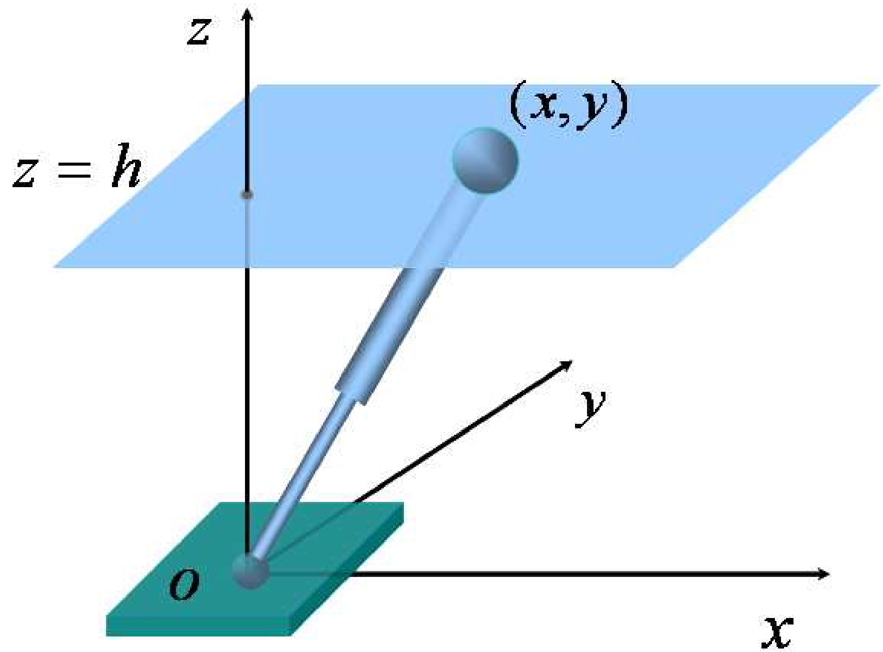
Stability

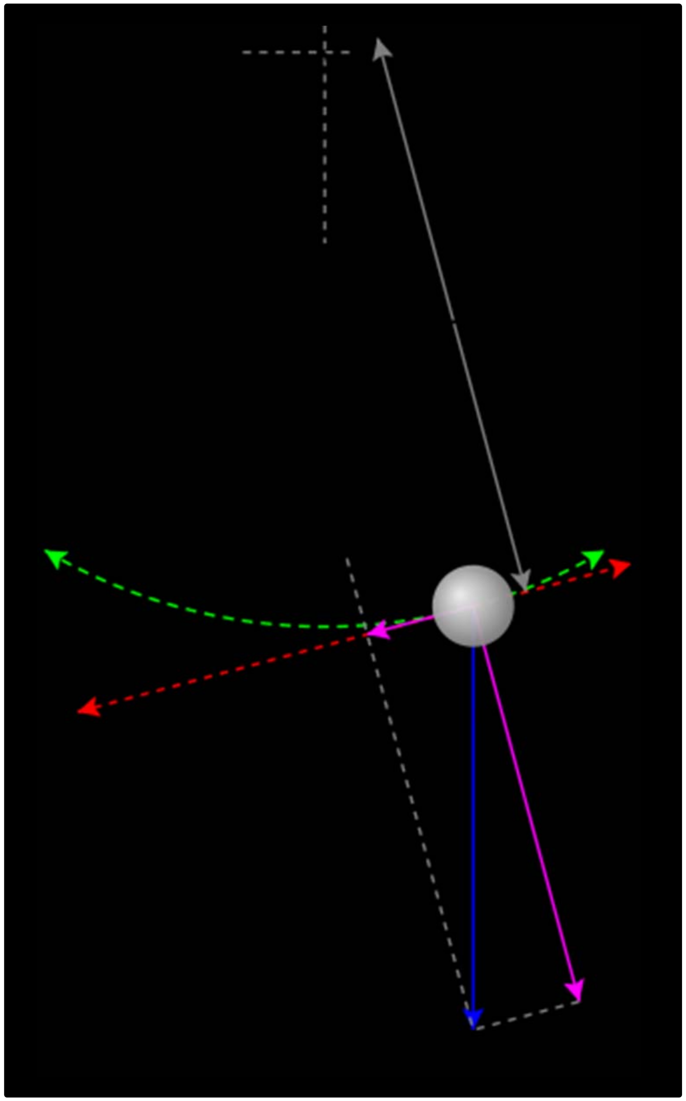
- Stability is the property of a system that determines if a system will return to an original state after a disturbance.
  - Stable: system returns to original equilibrium state after a disturbance



Stability

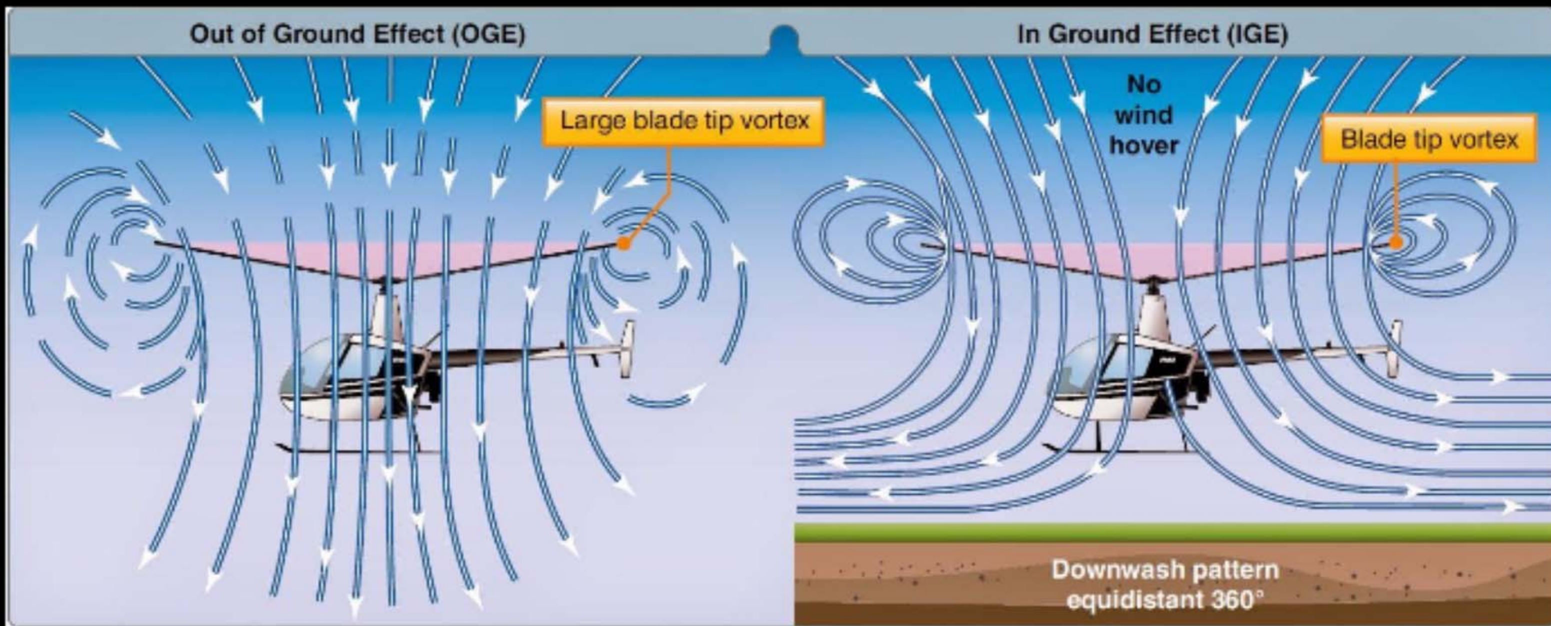
- Quadcopter can be modeled as an inverted pendulum, or a pendulum, based on the COG along the z-axis.





https://www.youtube.com/watch?v=8Ys2ixVBYXU

Ground Effect



In Ground Effect: Extra pressure occurs below the propeller

→ Causes a “ram” effect, improving the thrust

→ Can cause bouncing near the ground, using “air cushion”

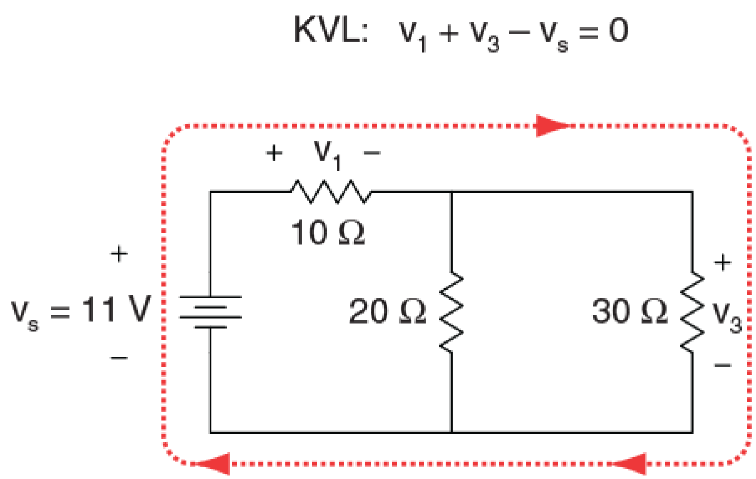
https://www.youtube.com/watch?v=DWDdh1-B4Io\

Electric Circuit Analysis

- Learning the language of circuits
- Ohm's Law and Kirchhoff's Law
- Basic circuit analysis (series, parallel)

Kirchhoff's Voltage Law

- Conservation of Energy

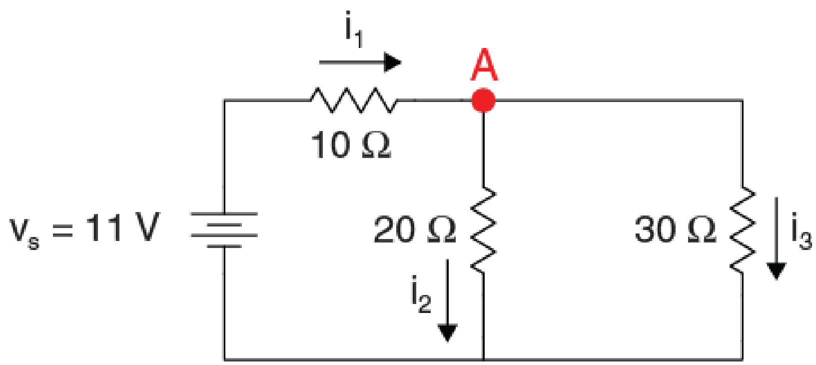


- Voltages sum to zero around any closed path
  - Can be applied to any closed loop in circuit
    - Pay attention to signs and polarity!

Kirchhoff's Current Law

- Conservation of charge

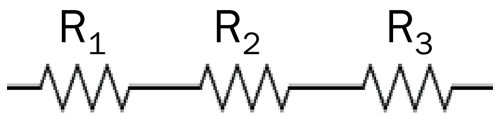
KCL A: $i_2 + i_3 - i_1 = 0$



- Currents coming out of any part of circuit must sum to zero
  - Can be applied at any point (node) in circuit
    - Pay attention to signs and polarity!

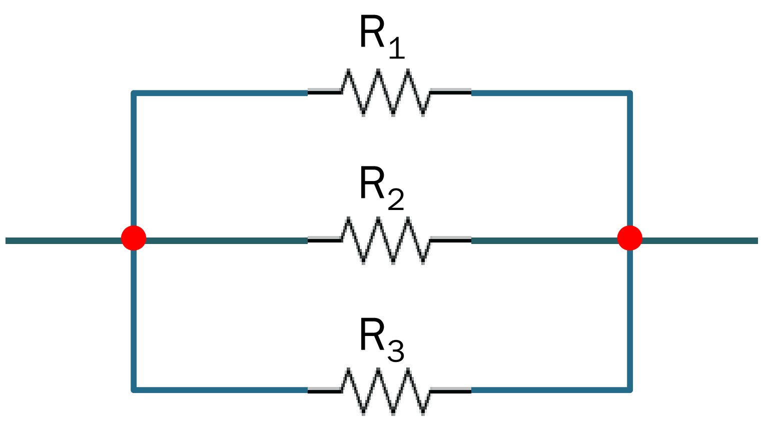
Resistors in Series and Parallel

- Series



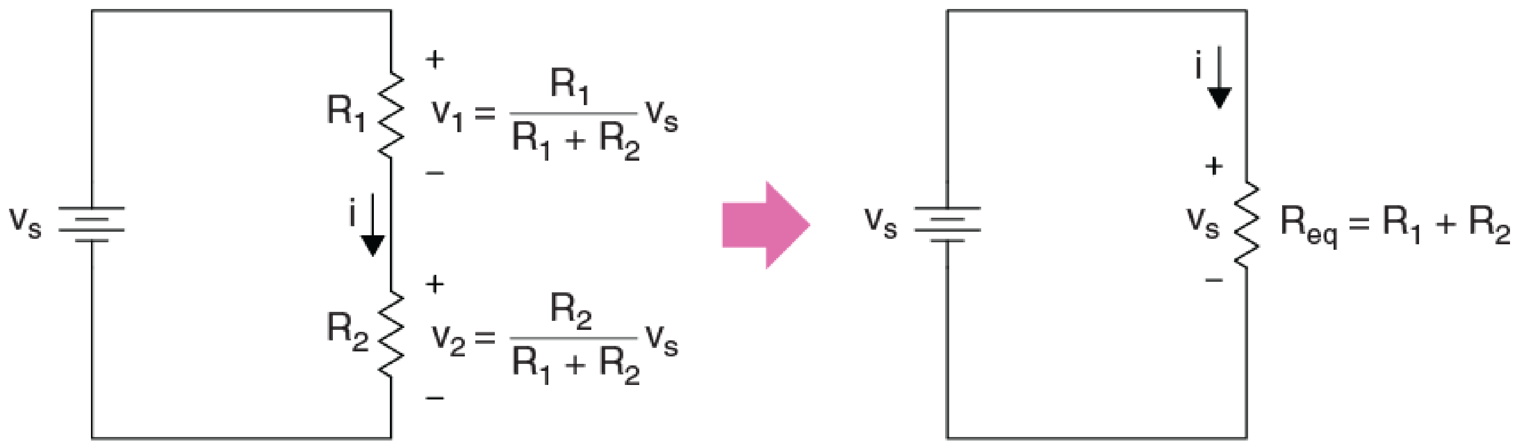
$$R_{eq} = R_1 + R_2 + R_3$$

- Parallel



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Series (Voltage Divider)



KCL: $i_1 = i_2 \;\; (= i)$

KVL: $v_s = v_1 + v_2$

$= i(R_1 + R_2)$

WHEN RESISTORS ARE CONNECTED IN SERIES:

- Equivalent resistance: $R_{eq} = R_1 + R_2$
- Currents are the same in R_1, R_2 and R_{eq}
- Voltage divides in proportion to R

Parallel (Current Divider)

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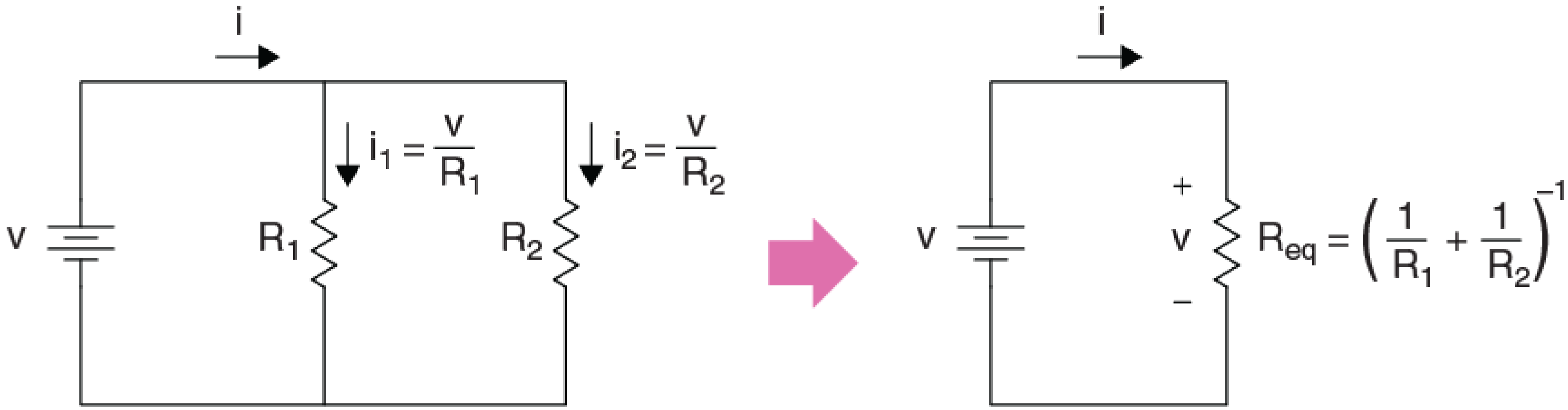


KVL: $v_1 = v_2 = v$

KCL: $i = i_1 + i_2$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v$$

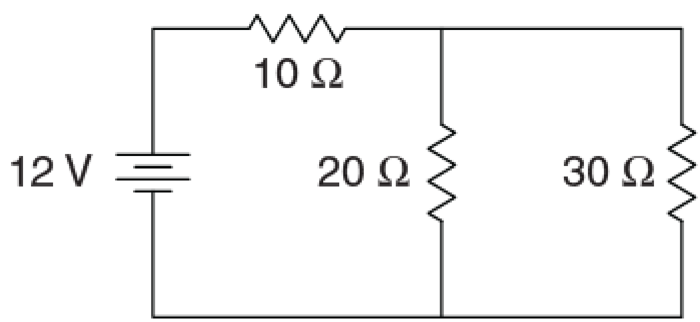
$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}\, i \qquad i_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}\, i$$

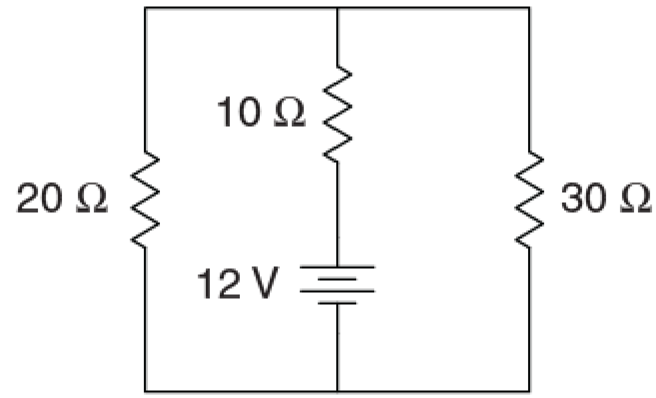
WHEN RESISTORS ARE CONNECTED IN PARALLEL:

- Equivalent resistance: $1/R_{eq} = 1/R_1 + 1/R_2$
- Voltages are the same in R_1, R_2 and R_{eq}
- Current divides in proportion to 1/R

Circuit Schematics

- Show how things are connected



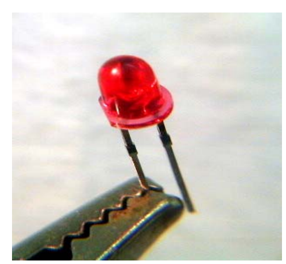


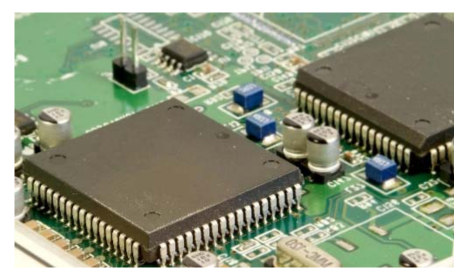
  - Many equivalent ways to draw the same circuit
  - Length of wire (line) has no meaning in schematic diagram
  - Circuit schematic does not show:
    - Where the components are physically placed on circuit board
    - Relative orientation of elements
    - Location of solder joints or connections

Electronic Components

- Passive Components
  - Resistors
  - Capacitors
  - Inductors
  - Diodes
  - Interface components
- Circuits
  - Kirchhoff’s Laws
  - Series
  - Parallel

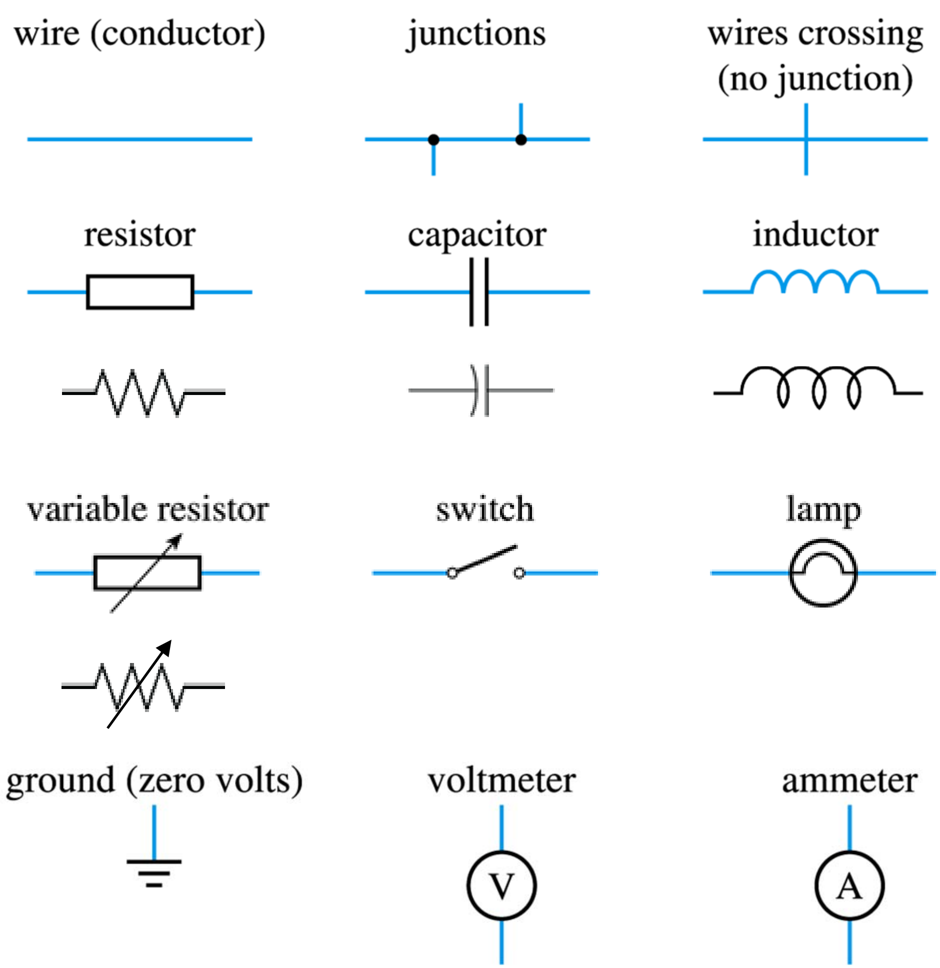
- Active Components
  - Transistors
  - Integrated circuits
    - Analog
    - Digital
    - Microcontroller (e.g. Arduino)

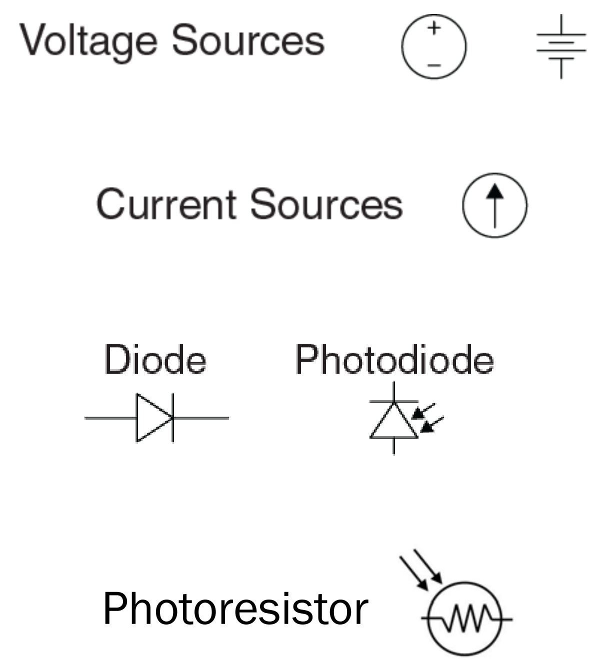




Electronic Components

- Symbols





Example

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- *A Practical Example: Light up a LED with a 9V battery*

Example

- *A Practical Example: Light up a LED with a 9V battery*

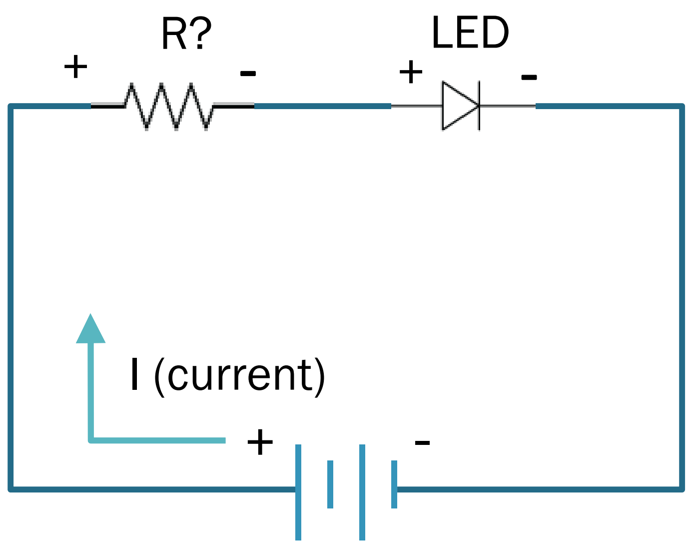
TYPICAL ELECTRICAL & OPTICAL CHARACTERISTICS (T_A = 25°C)

Characteristics	Color	Symbol	Condition	Unit	Minimum	Typical	Maximum
Forward Voltage	Red	V_F	I_F = 20 mA	V		2.1	2.6
	Blue/Green	V_F	I_F = 20 mA	V		3.4	4.0
Reverse Current	Red	I_R	V_R = 5 V	µA			100
	Blue/Green	I_R	V_R = 5 V	µA			100
Dominant Wavelength	Red	λ_D	I_F = 20 mA	nm	619	621	624
	Green	λ_D	I_F = 20 mA	nm	520	527	535
	Blue	λ_D	I_F = 20 mA	nm	460	470	475
Luminous Intensity	C5SMF - Red	I_V	I_F = 20 mA	mcd	1100	2200	
	C5SME - Red	I_V	I_F = 20 mA	mcd	770	1100	
	Green	I_V	I_F = 20 mA	mcd	2130	4400	
	Blue	I_V	I_F = 20 mA	mcd	550	1100	

http://www.digikey.com/product-detail/en/C5SMF-RJS-CT0W0BB2/C5SMF-RJS-CT0W0BB2CT-ND/1987481

Example

- *A Practical Example: Light up a LED with a 9V battery*



Battery, V = 9.35V (measured)
Operating Voltage of LED:
V_{LED} = 2.1 V

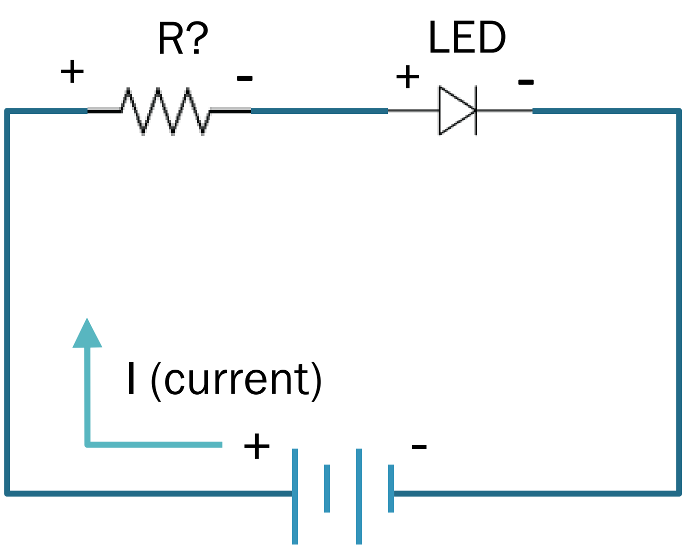
LED Steady current:
I = 20mA

Which resistor to use?

*Resistor doesn't have polarity, the + - signs are used to show the direction of voltage drop

Example

- *A Practical Example: Light up a LED with a 9V battery*



$V_R = V - V_{LED}$

$V_R = 9.35V – 2.1V = 7.25V$

$I = 20\ mA = 0.02A$

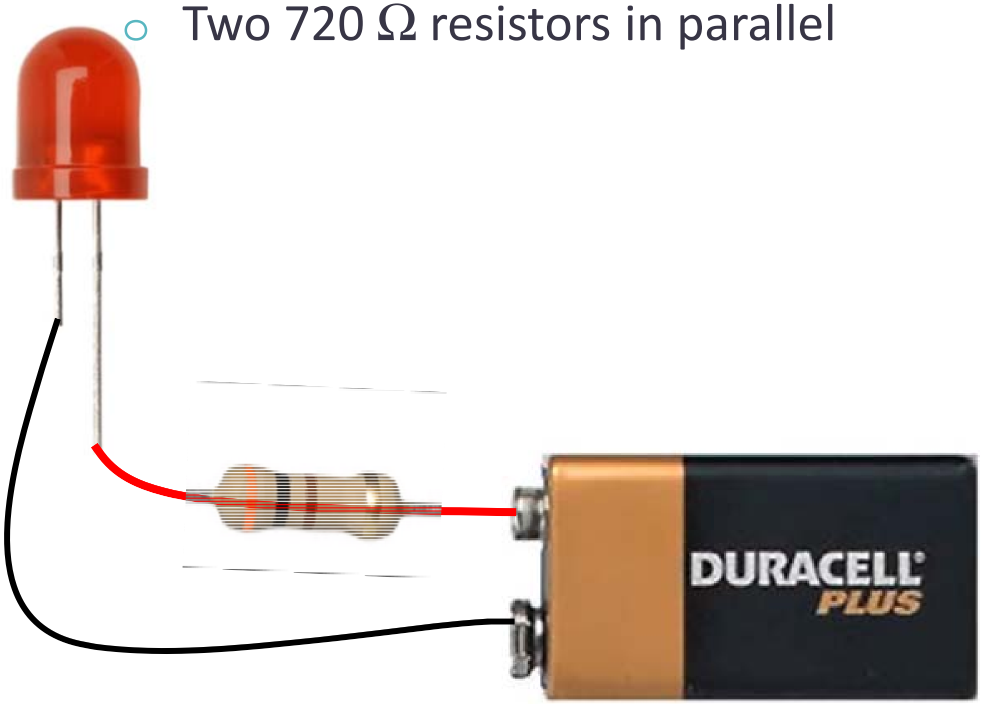
$R = V/I = 7.25V/0.02A = 362.5\ \Omega$

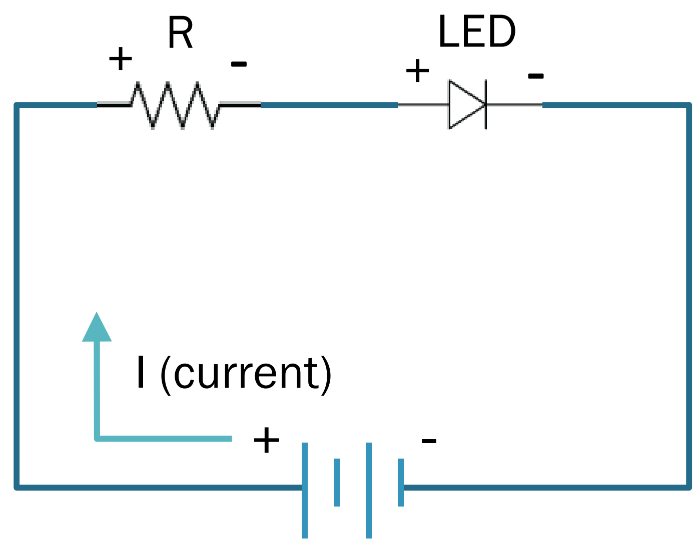
$R_{LED} = \underbrace{V/I}_{\text{Total resistance}} – R = 105\ \Omega$

Total resistance 467.5 Ω

Example

- *DEMO: Light up a LED with a 9V battery*
  - *One 360 Ω*
  - *Two 180 Ω* resistors in series
  - Two 720 Ω resistors in parallel

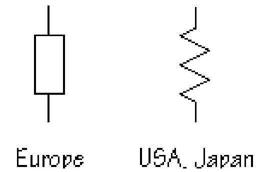




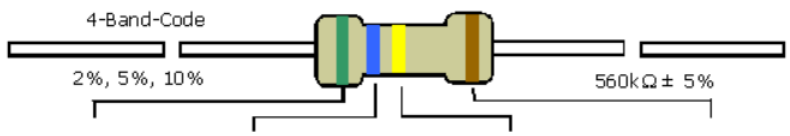
Convention:
Black wire for ground

Electronic Components

- Resistors



Resistor Color Code Guide



COLOR	1st BAND	2nd BAND	3rd BAND	MULTIPLIER	TOLERANCE
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	±0.5% (D)
Blue	6	6	6	1MΩ	±0.25% (C)
Violet	7	7	7	10MΩ	±0.10% (B)
Grey	8	8	8		±0.05%
White	9	9	9		
Gold				0.1	± 5% (J)
Silver				0.01	± 10% (K)

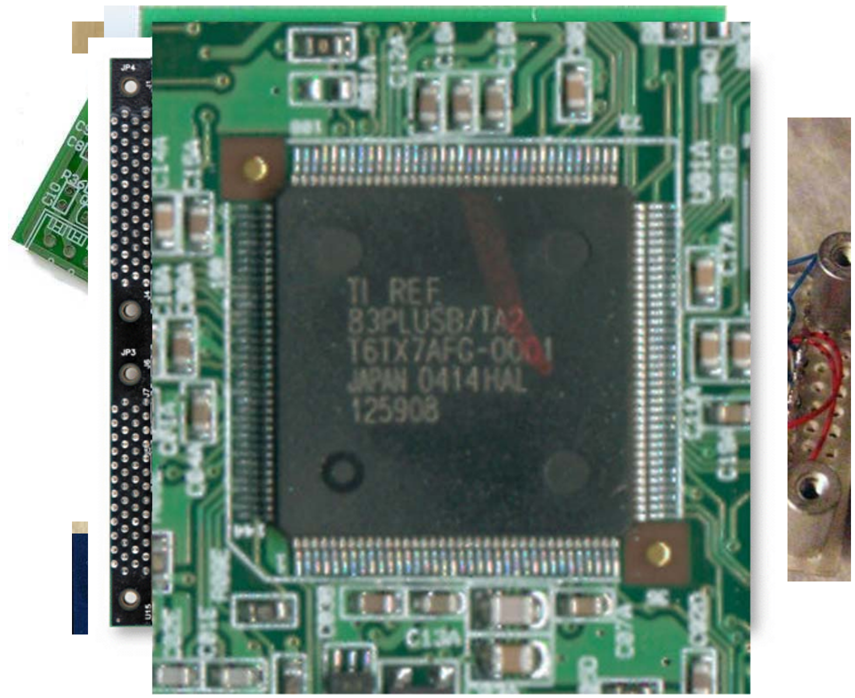
http://www.resistorguide.com/

Or Measure with multimeter

Building Circuits

- From Prototype to Product
  - Solderless breadboard
  - Soldered perfboard
  - Wire wrapping
  - Printed circuit board (PCB)
  - Microprocessor / DSP / FPGA
  - ASIC (application-specific integrated circuit)

Images taken from:
http://itp.nyu.edu/physcomp/Tutorials/
http://www.sas.org/
http://www.home.aone.net.au/
http://arduino.cc/
http://www.innovative-dsp.com/
http://www.datamath.org/
http://prt.fernuni-hagen.de/



Important Items in Lab

- Open Circuit and Short Circuit

Open circuit ⟺ No current flow between A and B

Short circuit ⟺ No voltage drop between A and B

Do NOT Short Circuit

Battery Energy

Total Capacity: 3000 mA-h

$$Energy\ [mA-h] = Current[mA] * Time[hrs]$$

3000 mAh = 3Ah

→ Will run 1 hour on 3 amp current draw

→ 30 minutes on 6 amp current draw

<u>SunnySky 1500KV</u>
Max Current / Motor = 20 A

Full Throttle Flight Time

$$= \frac{3Ah}{20A * 4} = 0.0375\ hrs = 2.25\ mins$$

Reading Assignment for Week 6

"Introduction to Engineering Design" Book 11

Engineering Skills and Quadcopter Missions 4th Edition 2017

Chapter 3 "Flight Dynamics" Sections 3.6 until the end of the chapter

Announcement

- Due to Veteran’s Day, next week’s lecture is moved to **Wednesday 11/14/2018** to 5 pm (EH 1200)