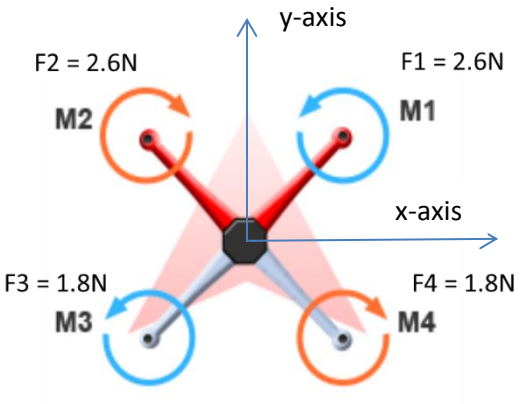
Homework 2: Moment, Center of Gravity and Thrust

The due date is October 26 (Friday) by 5pm. Please deposit it in the slot of the corresponding cardboard box in the room ET 408. Write or print neat solution and staple sheets together. Please write your name, ID number and lab session.



1. See the quadcopter configuration to the right. The forces generated by four motors (M1, M2, M3, M4) are listed in the picture with the red arrow indicating the front of the quadcopter. The motor to motor distance is 12 inches. The x and y axes are labeled in the positive direction, and the z-axis is positive going out of the plane of the paper.
   a. Identify which axis the quadcopter will rotate around.
   b. Determine whether this motion is considered roll, pitch, or yaw.
   c. Determine the moment/torque generated. Is the generated moment in the positive or negative direction?

Answer:

a. The quadcopter will rotate around X-axis

b. Pitch

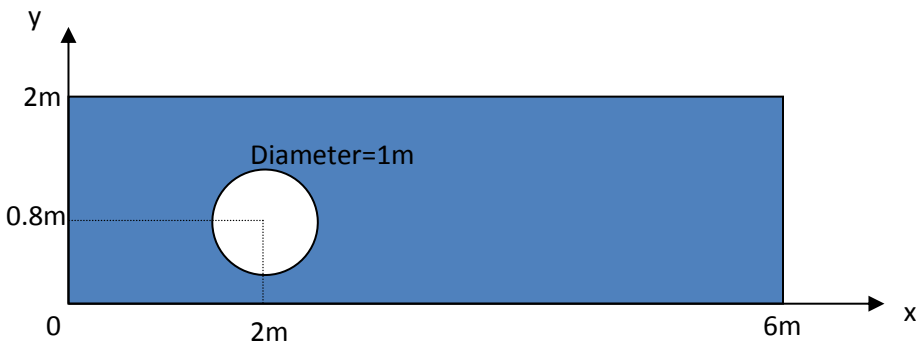
c. $\boldsymbol{M = \sum M = (F1 + F2)(d) - (F3 + F4)(d)}$ $\boldsymbol{d = \sqrt{\frac{(6in)^2}{2}} = 4.2426in = 0.1078m}$

$\boldsymbol{M = (2.6N + 2.6N)(0.1078m) - (1.8N + 1.8N)(0.1078m) = 0.1725Nm}$

Counter Clockwise, Positive

2. Calculate the center of gravity for the shape drawn below. If necessary, you can assume thickness of the shape is T, density of material is ρ.

Hint: you can verify your calculation with Solidworks.



Answer:

$$X_{cg} = \frac{\sum X_{ig}\rho T A_i}{\sum \rho T A_i} = \frac{\sum X_{ig} A_i}{\sum A_i} \quad , \quad Y_{cg} = \frac{\sum Y_{iy}\rho T A_i}{\sum \rho T A_i} = \frac{\sum Y_{iy} A_i}{\sum A_i}$$

$$Xcg = \frac{(3m)(2m \cdot 6m) - (2m)(\frac{\pi}{4}(1m)^2)}{(2m \cdot 6m) - (\frac{\pi}{4}(1m)^2)} = 3.07m$$

$$Y_{cg} = \frac{(1m)(2m \cdot 6m) - (0.8m)(\frac{\pi}{4}(1m)^2)}{(2m \cdot 6m) - (\frac{\pi}{4}(1m)^2)} = 1.014m$$

3. A propeller with two blades has a diameter of 8 inches, a pitch of 20 degrees, and a chord of 0.75 inches. The motor has a maximum angular velocity of 6750 RPM's
   a. Determine the thrust and power created by the propeller. Units should be in Newtons and Watts respectively.
   b. What is the pressure difference, in Pascals, created by the propeller disc?
   c. Will a quadcopter with these motors and propellers be able to lift 1kg?

Answer:

a. Given:

$N = 2,\ R = 4in,\ \theta = 20°,\ c = 0.75in,\ \Omega = 6750\ \frac{rev}{min}$

Convert units:

$R = 0.1016m,\ c = 0.0191m$

$\theta = 20° \times \frac{2\pi rad}{360°} = 0.3491\ rad$

$\Omega = 6750\frac{rev}{min} \times \frac{2\pi rad}{1rev} \times \frac{1min}{60s} = 706.858\frac{rad}{s}$

Solve:

$A = \pi R^2 = \pi(0.1016m)^2 = 0.0324m^2$

$s = \frac{Nc}{\pi R} = \frac{2(0.0191m)}{3.14159(0.1016m)} = 0.1194$

$$T = 2\left(\frac{sa}{16}\left[\sqrt{1 + \frac{64}{3sa}\theta} - 1\right]\right)^2 \rho(\Omega R)^2 A$$

$$= 2\left(\frac{(0.1194)(5.7)}{16}\left[\sqrt{1 + \frac{64}{3(0.1194)(5.7)}0.3491rad} - 1\right]\right)^2 \left(1.1839\frac{kg}{m^3}\right)\left(\left(706.858\frac{rad}{s}\right)(0.1016m)\right)^2 (0.0324m^2)$$

$$= 4.3203N$$

$$P = 2\left(\frac{sa}{16}\left[\sqrt{1+\frac{64}{3sa}\theta}-1\right]\right)^3 \rho(\Omega R)^3 A$$

$$= 2\left(\frac{(0.1194)(5.7)}{16}\left[\sqrt{1+\frac{64}{3(0.1194)(5.7)}0.3491rad}\right.\right.$$

$$\left.\left.-1\right]\right)^3 \left(1.1839\frac{kg}{m^3}\right)\left(\left(706.858\frac{rad}{s}\right)(0.1016m)\right)^3 (0.0324m^2)$$

$$= 32.4059W$$

b. $T = A(p_L - p_U)0.1078$

$$(p_L - p_U) = \frac{T}{A} = \frac{4.3203N}{0.0324m^2} = 133.221Pa$$

c. $T = 4.3203N$

$$T = 4.3203kg\frac{m}{s^2} \times \frac{1}{9.8\left(\frac{m}{s^2}\right)} \times \frac{1000g}{1kg} = 440.844g$$

4 motors:

$$440.844g \times 4 = 1763.37g$$

Yes, these motors and propellers used on a quadcopter will lift 1kg of mass.