Dynamic Systems Based Analysis in Neurophysiology

BME 233: Dynamic Systems with Applications to Biology and Medicine,
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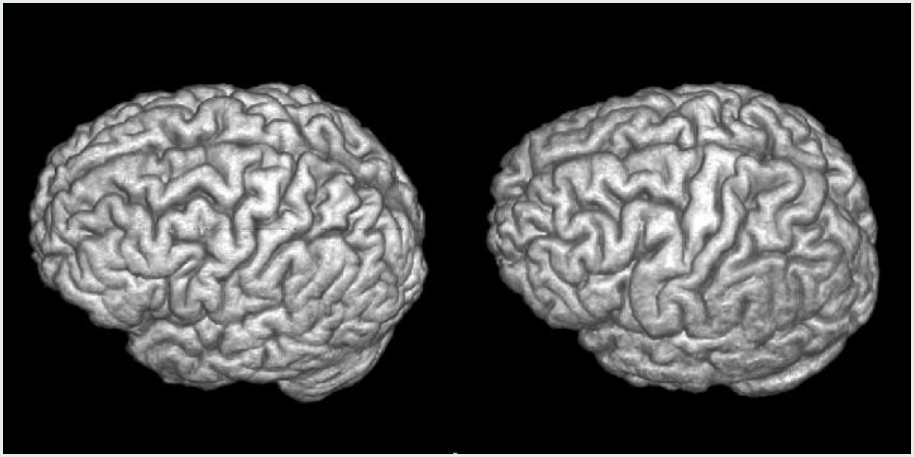
1. Why is human neuroanatomy difficult?

Subjects have an individual neuroanatomy. Even major structures show considerable structural variation - making automatical identification a hard problem. More recent parts of the brain (in terms of evolution) show a higher structural variability across individuals.

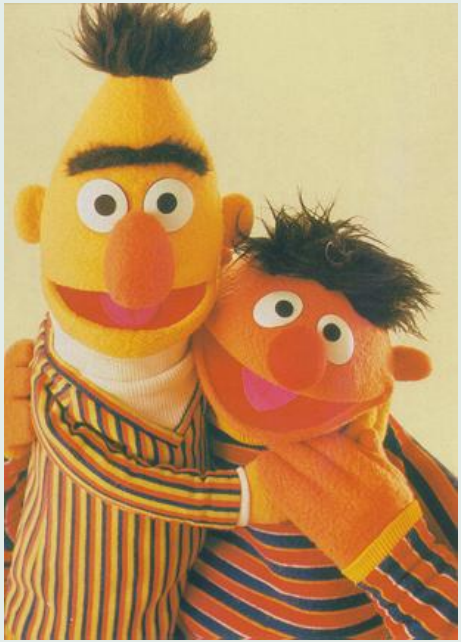
Below are surface renderings of aMRI data obtained in a pair of monozygotic twins.



Understanding the functional organization of the brain has to take individual structural differences into account.

2. Why is functional neuroimaging difficult?

- The signal-to-noise ratio is low.
- Methods have either a high temporal resolution (EEG) or a high spatial resolution (fMRI), both not both.
- The observed signal is an indirect correlate of the neuronal activity.
- The brain is a parallel processor. The induced task is just one of many currently active.
- Functional networks may be implemented differently in subjects.
- Subjects have different skills and pursue different strategies. Mood and actual performance play a role.

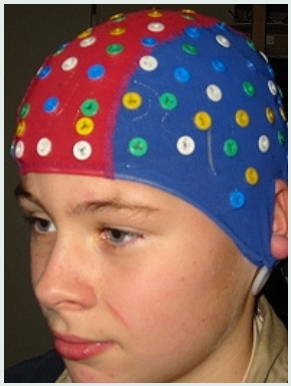


Event-related potentials

Correlates of the electrical activity of neurons in the brain can be measured on the scalp using electrodes and a low-noise amplifier. The recorded signal is called ”electroencephalogram” (EEG).

Many processes are active at any given time in the brain. In order to learn about the brain network that is involved in a specific task, this task has to be repeated several times (e.g., a few hundred times). The result is averaged, and activity that is not task-related cancels out.

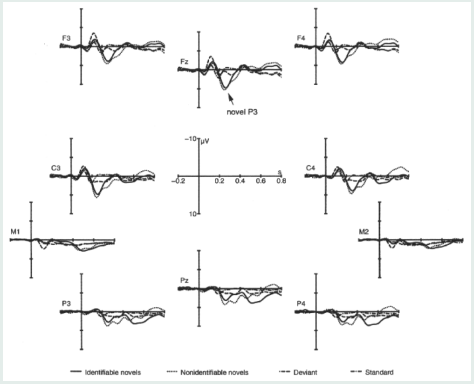
The resulting averaged signal is called ”event-related potential” (ERP).

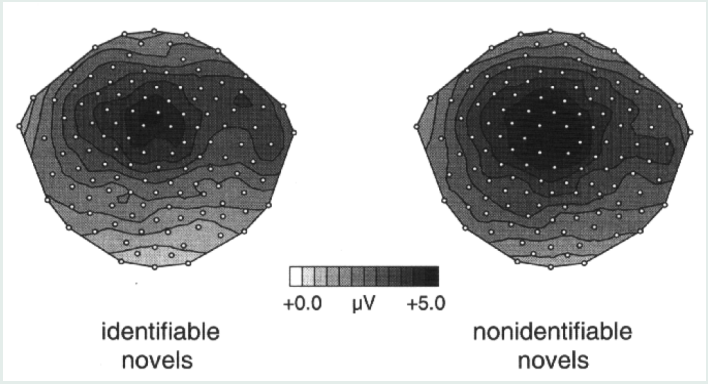


Example ERP study

Sounds were aurally and randomly presented to subjects while recording their EEG: (1) a pure sine tone of 600 Hz (standard condition), (2) a pure sine tone of 660 Hz (deviant condition), and (3) unique environmental sounds (novel condition).

Novel events, deviating from an ongoing auditory environment, elicit a positive event-related potential (ERP), the novel P3. Results are displayed either as a time-course per electrode position (left) or a spatial map (right).





Modeling the ERP

One expects that measurements $\mathbf{y}(t)$ can be expressed as a combination of different field maps $\mathbf{v}_i$ weighted by time-dependent factors x_i:

$$\mathbf{y}(t) = \sum_{i=1}^{n} x_i(t)\,\mathbf{v}_i.$$

Since information processing in the brain is based on interaction of local regions, amplitudes $x_i(t)$ are not independent of each other, but reflect interactions that may be expressed in terms of differential equations:

$$\frac{d}{dt}x_i = f_i(x).$$

A rigorous mathematical derivation for the emergence of low-dimensional dynamics in high-dimensional systems can be found in [Haken 1983, 1987]. For applications of this concept to brain dynamics, refer to [Kelso 1995, Nunez 1995].

Model setup

So the goal is to decompose an ERP signal $\mathbf{y}(t)$ into time-dependent amplitudes $x_i(t)$ and spatial field distributions $\mathbf{v}_i$.

An approximation of the spatio-temporal model may be obtained by minimizing a cost function C, consisting of a least-squares fit for the signal representation and the dynamics representation:

$$C = C_s + C_d = \frac{< \left(\mathbf{y}(t) - \sum_{i=1}^{n} x_i(t)\mathbf{v}_i\right)^2 >}{< \mathbf{y}(t)^2 >} + \sum_{i=1}^{n} \frac{< (x_i - f(x_i))^2 >}{< (x_i(t))^2 >},$$

where brackets denote a summation over time. Dynamics are modeled as polynomial functions:

$$\frac{d}{dt}x_i = f_i(x) = \sum_{j}^{n} a_{i,j}x_j + \sum_{j}^{n}\sum_{k=j}^{n} a_{i,j,k}x_j x_k + \sum_{j}^{n}\sum_{k=j}^{n}\sum_{l=k}^{n} a_{i,j,k,l}x_j x_k x_l.$$

Choosing polynomials here is not stringent, e.g., in the case of periodic observables, trigonometric basis functions may be considered.

Model adaptation

To reduce ambiguities in the solution space (e.g., $x_i(t) \sim x_j(t), i \neq j$), we constrain the amplitudes:

$$< x_i(t)\, x_j(t) >= \partial_{ij}$$

Further, we introduce biorthogonal modes u_i, so that $u_i v_j = \partial_{ij}$. Then, the unknown amplitudes can be determined by projecting the signal onto these modes:

$$x_i(t) = u_i \mathbf{y}(t).$$

So the task is to estimate model parameters $\{u_i, a_{i,j}, a_{i,j,k}, a_{i,j,k,l}\}$. Since we are considering low-dimensional signals ($n \in \{2, \ldots, 5\}$), between 10-50 parameters have to estimated. Genetic algorithms are the method of choice here.

System analysis

To gain more insight into the dynamics and to allow an interpretation by means of interacting processing units, the dynamical system has to be analyzed.

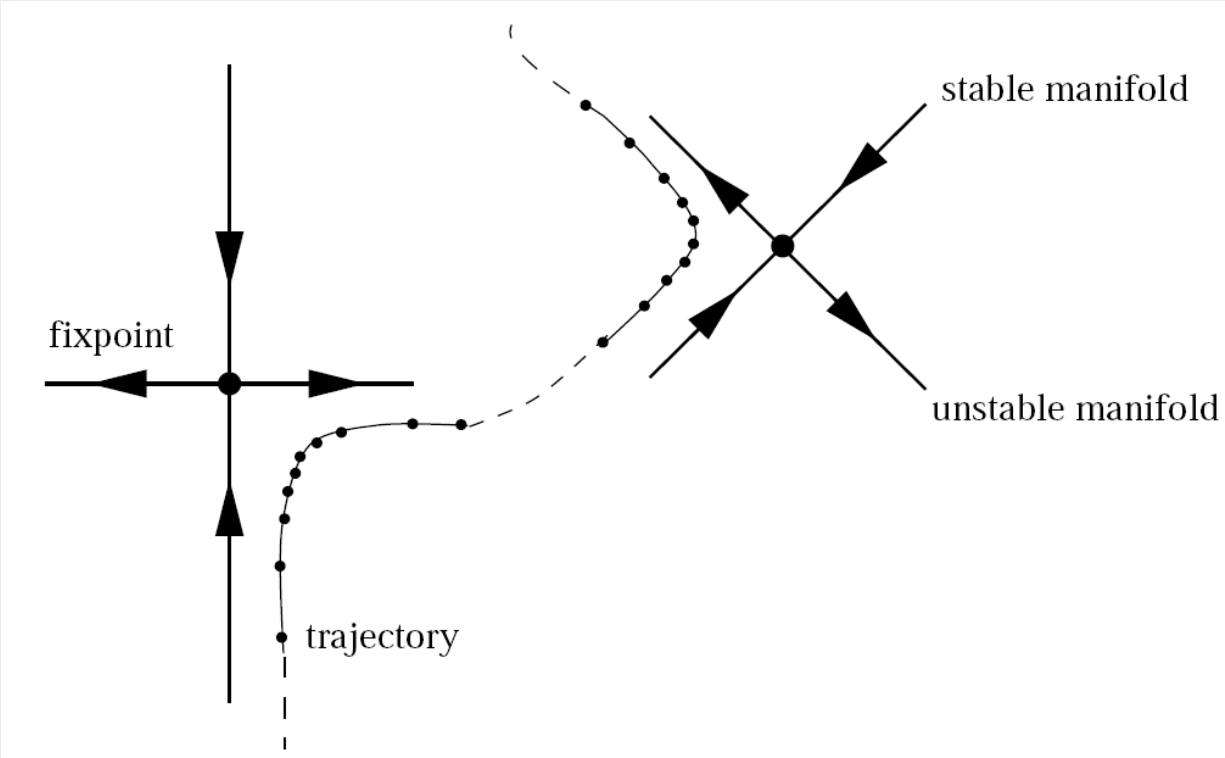
Dynamical systems can be characterized by stationary points, so-called fixed points, which attract or repel trajectories in the space of the amplitudes. These points do not evolve with time and are obtained by solving the non-linear set of equations:

$$f_i(x) = 0.$$

Note that fixed points are never reached by a trajectory, as it would stay there forever. Identifying these points and observing the trajectory being attracted in the course of time leads to a description of brain dynamics in terms of ”successive” processing states.

Signal trajectory

Trajectory passing saddle points. Dots represent data points.



Results for example study

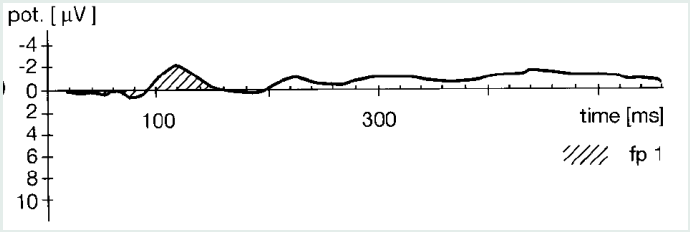
The ERP experiment described above was analyzed using $n = 2$ interacting modes. So we have two spatial field maps $\mathbf{v}_1, \mathbf{v}_2$ and two time-dependent amplitudes x_1, x_2 that explain the signal $\mathbf{y}(t)$:

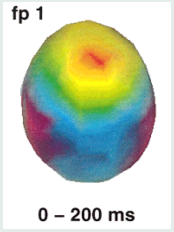
$$\mathbf{y} = x_1 v_1 + x_2 v_2.$$

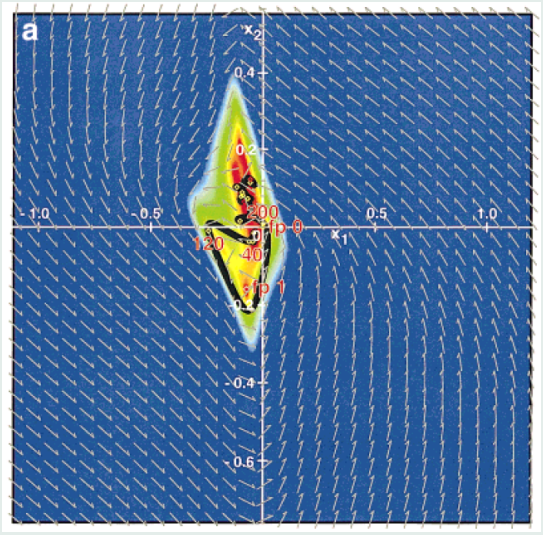
About 80-90% of the signal's variance was explained by the model, using $n = 3$ modes yields a representation of 95-97%. Models were computed for each experimental condition separately. In the following slides, the time course y at electrode Fz, the phase portrait of x_1 vs. x_2 and the spatial field maps corresponding to the fixed points are shown.

Results: standard condition

For the standard condition, a single fixed point is found, around which the trajectory cycles in the time window between 100-200 ms after stimulus onset, corresponding to an activation of the primary auditory cortex (N100 component):

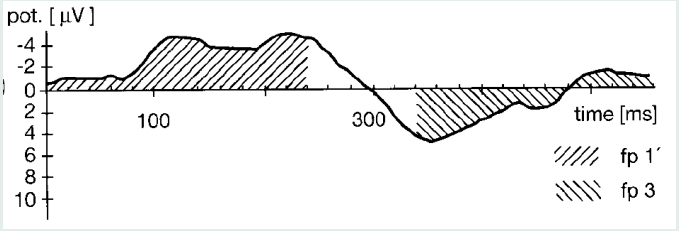


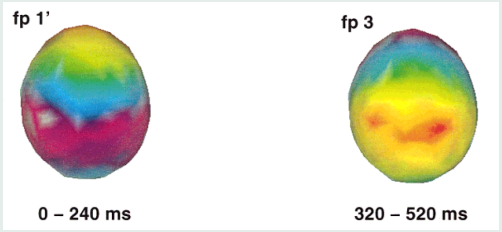


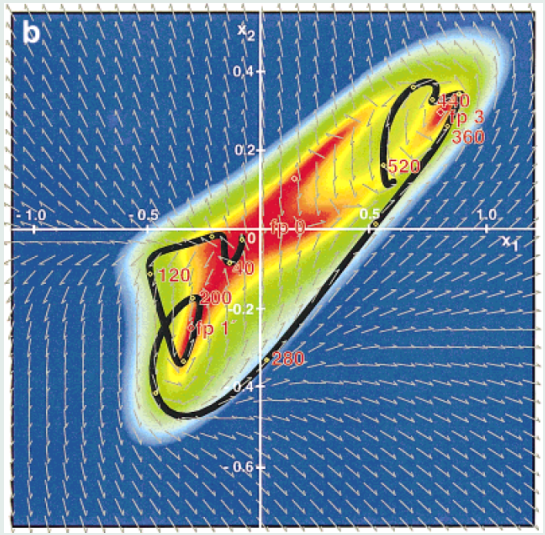


Results: deviant condition

For the deviant condition, a second fixed point in the time window between 340-500 ms is found (P300 component):

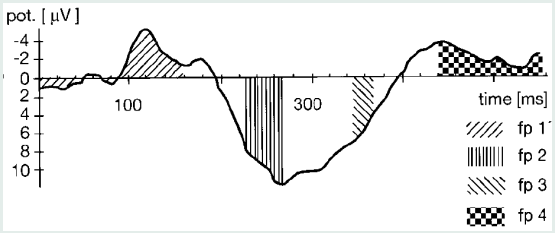


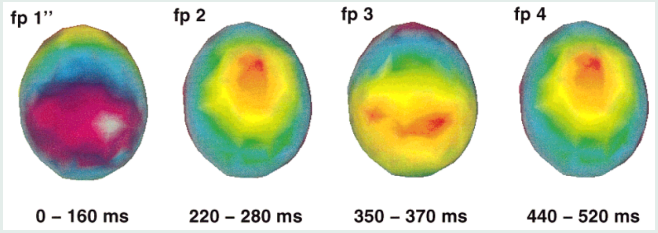


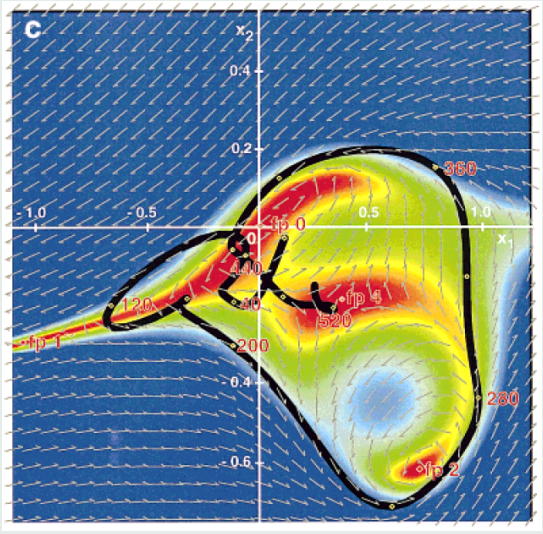


Results: novel condition

For the novel condition, the trajectory reaches a second fixed point in the interval 230-270 ms, then passed the fp corresponding to the P300 component at 360 ms, and is finally attracted by fp 4 at 480-520 ms:







Clustering approach

Note that phase points accumulate close to fixed points if the signal is sampled at a constant rate. Thus, the detection of stable manifolds in multidimensional signals can be treated as a recognition of point clusters in data space.

The fuzzy c-means algorithm was used to detect regions in data space with high density of data points. Note that we perform clustering in high-dimensional space now. A projection is not required anymore.

Model setup

An n-dimensional spatio-temporal signal $\mathbf{y}(t)$ is comprised of data points $y_s(t_i)$ at detection channel s and time point i. The algorithm determines C cluster centers k_c, whose Euclidean distance $d(y(t_i), k_c)$ to data points $y(t_i)$ is minimal. Distances are weighted by u_{ci}^m with $0 \leq u_{ci} \leq 1$ that indicate a degree of membership of data point i to cluster c. The exponent m is called "fuzziness factor". The cost function is written as:

$$J_m = \sum_{c=2}^{C} \sum_{i=1}^{T} u_{ci}^m \|y(t_i) - k_c\|^2 \quad \text{with} \quad \sum_{c=2}^{C} u_{ci}^m = 1,$$

where T denotes the number of time points, and $C \geq 2$ the number of clusters.

Model adaptation

We use a variant of the expectation maximization (EM) algorithm to find the cluster centers. First, we assume cluster centers k_c and estimate membership values:

$$u_{ci} = \left[\sum_{j=2}^{C} \left(\frac{\|y(t_i) - k_c\|^2}{\|y(t_i) - k_j\|^2} \right)^{1/(m-1)} \right]^{-1}.$$

Then, we update cluster centers:

$$k_c = \frac{\sum_{i=1}^{T} u_{ci} y(t_i)}{\sum_{i=1}^{T} u_{ci}^m}.$$

Both steps alternate until convergence. From a map of the distance of a data point $y(t_i)$ to the next cluster center k_c, a number of *temporal windows* can be defined, corresponding to the influence of a fixed point on the trajectory.

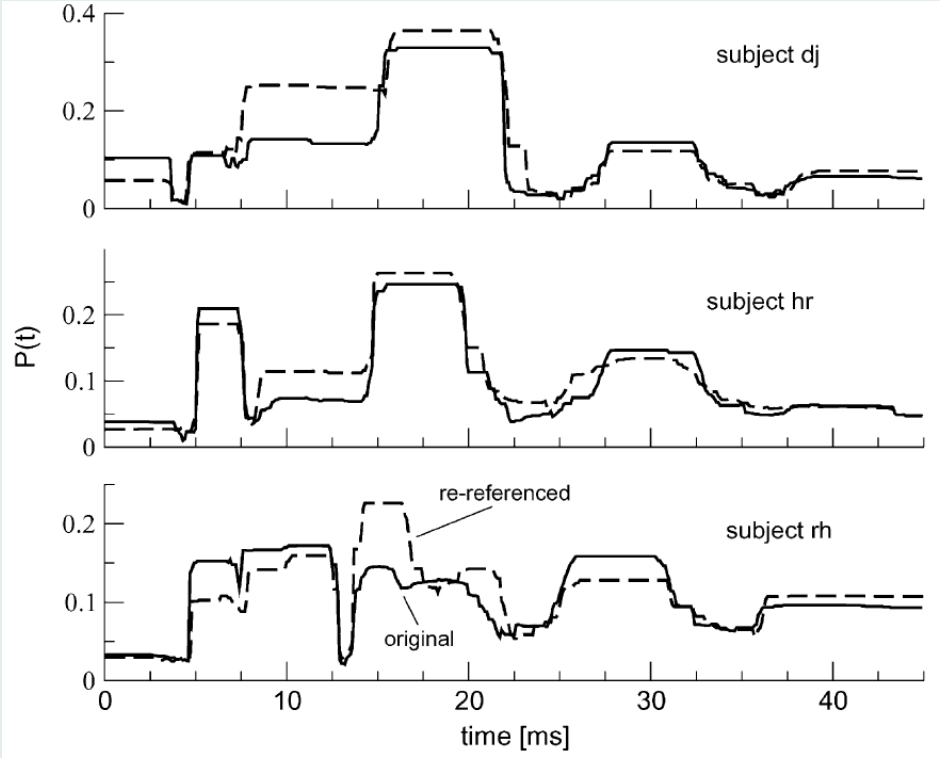
ERP experiment 2

Middle latency auditory evoked potentials were acquired at a sampling rate of 10 kHz at 32 electrode positions. Ten thousand trials were recorded at an average stimulation rate of 15 Hz. Raw data were band-pass filtered between 20-2000 Hz, and averaged across trials. Results are shown for three subjects.

Source: Hutt A., Riedel H., Physica D 177 (2003), 203-232.

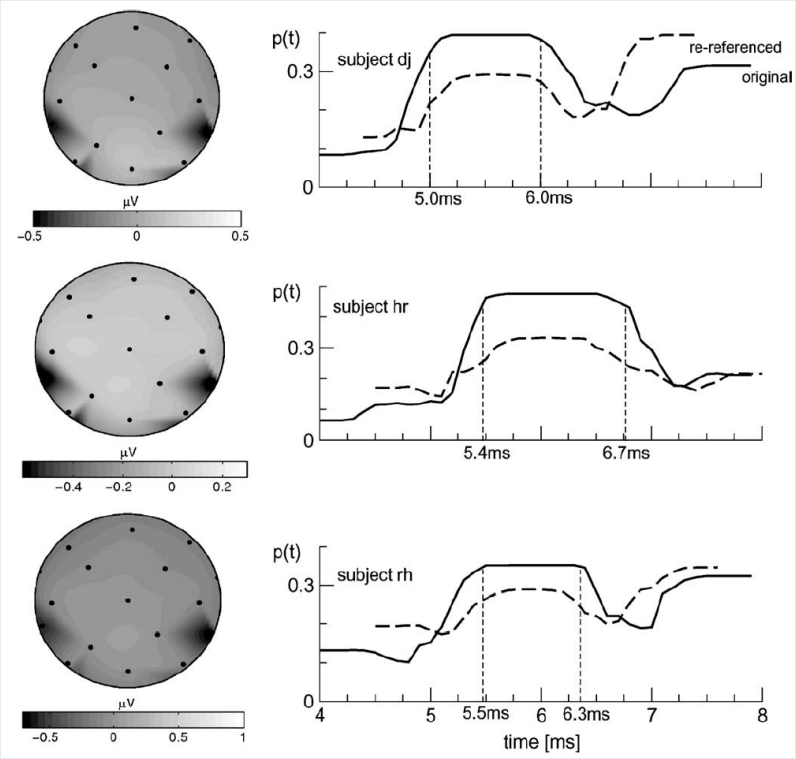
Cluster quality

Cluster quality for the three subjects. Clusters are recognized at 5, 12, 18, and 30 ms.



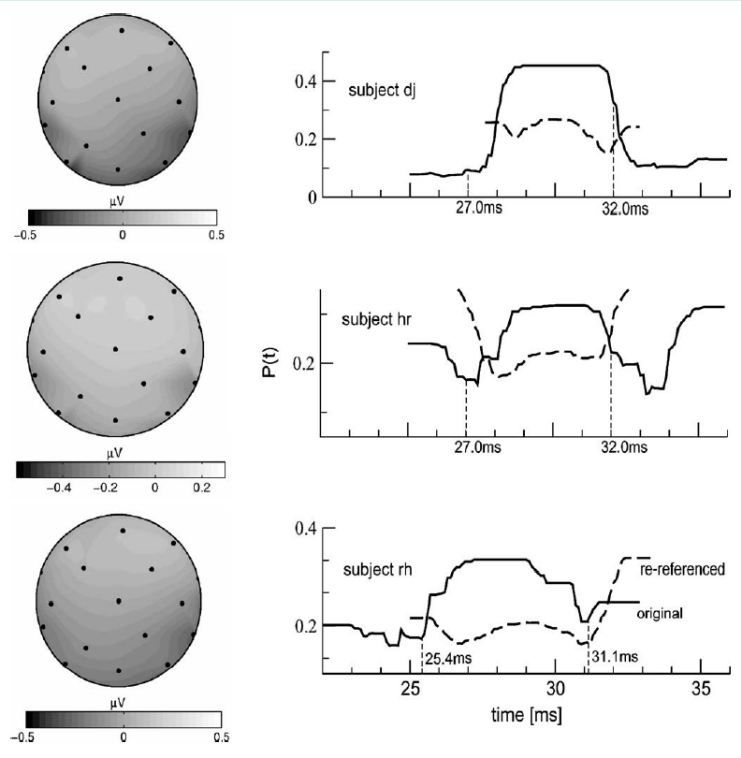
Time window I

Cluster results and spatial maps in the time window 4-8 ms.



Time window II

Cluster results and spatial maps in the time window 20-35 ms.



Summary

An analysis of neurophysiological data on the basis of dynamical systems theory was presented. This approach offers the following advantages over a ”classical” analysis:

- The ERP signal is analyzed concurrently in space and time.
- The signal trajectory is attracted and repelled by a series of fixed points in phase space.
- A data-driven method was proposed to determine the number of fixed points, their spatial potential configurations and their temporal windows.
- An ERP signal is understood as originating from a sequence of meta-stable potential configurations in the brain (that are low-dimensional), and high-dimensional transitions between them.

References

[1] Uhl C., Kruggel F., Opitz B., von Cramon D.Y. (1998) *A new Concept for EEG/MEG - Signal Analysis: Detection of Interacting Spatial Modes.* Human Brain Mapping 6, 137-149.

[2] Hutt A., Svensen M., Kruggel F., Friedrich R. (2000) *Detection of Fixed Points in Spatio-Temporal Signals by a Clustering Method.* Physical Review E 61, 4691-4693.

[3] Uhl C., Hutt, A., Kruggel F. (2001) *Improvement of Source Localization by Dynamical Systems Based Modeling (DBMS).* Brain Topography, 13, 219-226.

[4] Hutt A., Kruggel F. (2001) *Extension of Fixed-Point Clustering: A Cluster Criterion.* Nonlinear Sciences 2001, 1-5 (e-print arvix.org/abs/nlin.CD/0102032).

[5] Hutt A., Kruggel F. (2002) *Fixed-Point Analysis: Dynamics of Non-Stationary Spatio-Temporal Signals.* In: Boccaletti S., Mancini H.L., Gonzalez-Vinas W., Burguete J., Valladares D.L. (eds.), Space-Time Chaos: Characterization, Control and Synchronization, pp. 29-42. World Scientific, Singapore.

Publications may be found on the Lab's website (http://sip.eng.uci.edu/).