44715: Optimization II **2019S**

Homework 2: Constrained Optimization

Name: *ID:*

Note: *In all notes, bold face letters denote vectors.*

1. Use the Lagrangian to find local extremizers (maximum and minimum) for the following optimization problems.

   (a)
$$\begin{aligned} \text{minimize} \quad & x_1^2 + 2x_1x_2 + 3x_2^2 + 4x_1 + 5x_2 + 6x_3 \\ \text{subject to} \quad & x_1 + 2x_2 = 3 \\ & 4x_1 + 5x_3 = 6 \end{aligned} \tag{2.1}$$

   (b)
$$\begin{aligned} \text{maximize} \quad & x_1x_2 \\ \text{subject to} \quad & x_1^2 + 4x_2^2 = 1 \end{aligned} \tag{2.2}$$

   (c)
$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}\mathbf{x}^T A\mathbf{x} + \mathbf{b}^T\mathbf{x} \\ \text{subject to} \quad & B\mathbf{x} = \mathbf{c} \end{aligned} \tag{2.3}$$

   where $A \in \mathbb{R}^{n\times n}$ is positive definite, $B \in \mathbb{R}^{m\times n}$, $m \leq n$.

2. Find a point on the spherical surface $\|\mathbf{x}\| = 3$ with $\mathbf{x} \in \mathbb{R}^n$ such that this point has the smallest distance to the point $[1, 0, 0, \ldots, 0]^T$.

3. Consider the problem
$$\begin{aligned} \text{minimize} \quad & 2x_1 + 3x_2 - 4 \\ \text{subject to} \quad & x_1x_2 = 6 \end{aligned} \tag{2.4}$$

   (a) Use Lagrange condition to find all possible local extremizers.

   (b) Use the second order sufficient conditions to specify which points are strict local minimizers and which are strict local maximizers.

   (c) Are these local extremizers global?

4. Consider the optimization problem
$$\begin{aligned} \text{minimize} \quad & x_1^2 + 4x_2^2 \\ \text{subject to} \quad & x_1^2 + 2x_2^2 \geq 4 \end{aligned} \tag{2.5}$$

   (a) Final all points that satisfy the KKT conditions.

   (b) Apply the second order condition to determine the whether or not the local extremizers are local minimizers or maximizers or neither.

5. Consider the problem

$$
\begin{aligned}
\text{minimize} \quad & \frac{1}{2}\|A\mathbf{x} - \mathbf{b}\|^2 \\
\text{subject to} \quad & x_1 + x_2 + \dots x_n = 1 \\
& x_1, x_2, \dots, x_n \geq 0
\end{aligned} \tag{2.6}
$$

Write down the KKT condition for the above problem.