Homework to Chapter 29

Friday, November 8, 2019 7:25 PM

(1) Find the minimal polynomial of $\sqrt[3]{2+\sqrt{3}}$ over $\mathbb{Q}$
Why is it irreducible?

(2) Let $\alpha \in \mathbb{C}$ be transcendental over $\mathbb{Q}$ and $\beta \in \mathbb{Q}(\alpha) \setminus \mathbb{Q}$
Show that α is algebraic over $\mathbb{Q}(\beta)$

Hint Represent β as $\frac{f(\alpha)}{g(\alpha)}$ and use this to find a polynomial with coefficients in $\mathbb{Q}(\beta)$ that has α as a root

(3) Show that $\cos\left(\frac{2\pi}{5}\right)$ is algebraic over $\mathbb{Q}$. What is the degree of its minimal polynomial.

(4) Show that the fields $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic

(5) Let $\mathbb{Z}_p \subset E$ a field extension and assume that $\alpha \in E \setminus \mathbb{Z}_p$ is a root of $f(x) = x^2 - a$ with $a \in \mathbb{Z}_p$.
Classify $(\mathbb{Z}_p(\alpha)^*, \times)$ according to the Fundamental Theorem of Finitely Generated Abelian Groups

(6) Let $F \subset E$ be an extension of finite fields.
Suppose that $|F| = p^a$ and $|E| = q^b$ with p, q prime. Show that

(a) $p = q$
(b) a divides b.