

3A: Extra exercises 4

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 (6 points)

Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

- (a) Compute the reduced row echelon form of A . (2 points)
- (b) Find a basis of the null space of A . (1 point)
- (c) What is the dimension of the null space of A ? (1/2 point)
- (d) Find a basis of the column space of A . (1 point)
- (e) What is the rank of A ? (1/2 point)
- (f) Find all possible subsets of the columns of A which form a basis of the column space of A (tricky, 1 point).

Exercise 2 (4 points)

Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 3 & 1 & 2 \\ -1 & 1 & 1 & 0 \end{bmatrix}.$$

- (a) Compute the determinant of A using row reductions. (2 points)
- (b) Compute the determinant of A using cofactor expansions. (2 points)