
Remark: the exercises below will be graded carefully. Give explanations and computations.

Exercise 1 (5 points)
Let

$$A = \begin{bmatrix} 2 & 1 & 3 & 3 \\ 0 & 1 & 3 & 2 \\ 1 & 2 & 4 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}.$$

(a) Find the determinant of A (1.5 points).
(b) Show that the equation

$$A\mathbf{x} = [1, 0, 0, 1]^T$$

has a unique solution (0.5 points).
(c) Compute the solution to the equation in (b) using Cramer's rule (1.5 points).
(d) Compute the second column of the inverse of A using determinants (Theorem 8 in Section 3.3, 1.5 points).

Exercise 2 (5 points)
Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}.$$

(a) Compute the characteristic polynomial of A (1 point).
(b) Compute the eigenvalues of A (1 point).
(c) Use the results from (a) to answer the following question: is A invertible? (1 point)
(d) For reach of the eigenvalues, compute a basis of the corresponding eigenspace (2 points).