

### 3A: Extra exercises 6

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Remark: the exercise below will be graded carefully. Give explanations and computations.

#### Exercise 1

Let  $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$  be the linear map given by the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \\ 1 & 1 & 1 \end{bmatrix}.$$

Consider the basis

$$\mathcal{B} = \{[1, 1, 0]^T, [1, -1, 1]^T, [0, 2, -2]^T\}$$

of  $\mathbf{R}^3$ . Compute  $[T]_{\mathcal{B}}$ .

#### Exercise 2

Let  $\theta \in [0, 2\pi]$ . Consider the (rotation) matrix

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

over the complex numbers  $\mathbf{C}$ .

(a) Compute the eigenvalues of  $A$ .

(b) Find an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $A = PDP^{-1}$  over the complex numbers.