3A: Extra exercises 7

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1
Let A be the matrix

$$A = \begin{bmatrix} 0 & -2 & -2 & 1 & -1 \\ 0 & 2 & 2 & 1 & 3 \\ 0 & -1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 3 \end{bmatrix}.$$

Let $W = \mathrm{Col}(A)$ be the column space of A. Let $\mathbf{v} = [8, 0, -7, 1]^T \in \mathbf{R}^4$.
(a) Compute a basis for W.
(b) Compute an orthonormal basis of W.
(c) Compute the orthogonal projection $\mathrm{Proj}_W(\mathbf{v})$ of $\mathbf{v}$ on W.
(d) Compute the distance between W and $\mathbf{v}$.

Exercise 2
(a) Let $\mathbf{u}, \mathbf{v} \in \mathbf{R}^n$. Show that

$$\mathbf{u} \cdot \mathbf{v} \leq \|\mathbf{u}\| \|\mathbf{v}\|.$$

(Hint: expand $\|\mathbf{u} - c\mathbf{v}\|$ for a specific $c \in \mathbf{R}$).
(b) Prove that for $a, b, c \in \mathbf{R}$ one has

$$2a^2 + 3b^2 + c^2 \leq \sqrt{2a^2 + 9b^2 + c^2}\sqrt{2a^2 + b^2 + c^2}.$$

Exercise 3
Find an orthonormal basis of the null space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}.$$