3A: Extra exercises 2

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1

Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 3 & 6 & 0 & 3 & 1 \\ 2 & 4 & 2 & 1 & 1 \end{bmatrix}$ and vector $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Finally, consider the vector

$$\mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 3 \end{bmatrix}.$$

a) Compute $A\mathbf{c}$.
b) Find the solution(s) of the equation

$$A\mathbf{x} = \mathbf{b}$$

in parametric vector form.
c) Find the solution(s) of the equation

$$A\mathbf{x} = \mathbf{0}$$

in parametric vector form.
d) Determine whether the first, second and fourth columns of A are linearly independent or not.
e) Let $T : \mathbf{R}^5 \to \mathbf{R}^3$ be the linear transformation whose standard matrix is A. Is T one-to-one? Is T onto?

Exercise 2

Show that there is a unique linear map $T : \mathbf{R}^3 \to \mathbf{R}^3$ with

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

and compute the standard matrix A corresponding to this map.