3A: Extra exercises 4

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 (6 points)
Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Compute the reduced row echelon form of A. (2 points)
(b) Find a basis of the null space of A. (1 point)
(c) What is the dimension of the null space of A? (1/2 point)
(d) Find a basis of the column space of A. (1 point)
(e) What is the rank of A? (1/2 point)
(f) Find all possible subsets of the columns of A which form a basis of the column space of A (tricky, 1 point).

Exercise 2 (4 points)
Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 3 & 1 & 2 \\ -1 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Compute the determinant of A using row reductions. (2 points)
(b) Compute the determinant of A using cofactor expansions. (2 points)

Exercise 3 (4 points)
Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \ \mathbf{b_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \mathbf{b_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Let $\mathfrak{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$.
(a) Prove that $\mathfrak{B}$ is a basis of $\mathbf{R}^2$. (1 point)
(b) Compute $[A]_{\mathfrak{B}}$. (3 points; see Section 5.4)