Remark: the exercise below will be graded carefully. Give explanations and computations.

**Exercise 1** (10 = 2.5 + 1 + 0.5 + 1.5 + 1 + 1.5 + 1 + 1 points)Let

$$\mathbf{u}_{1} = \begin{bmatrix} 1\\1\\1\\1 \end{bmatrix}, \ \mathbf{u}_{2} = \begin{bmatrix} 1\\0\\0\\1 \end{bmatrix}, \ \mathbf{u}_{3} = \begin{bmatrix} 0\\2\\1\\-1 \end{bmatrix}, \ \mathbf{u}_{4} = \begin{bmatrix} 2\\3\\2\\1 \end{bmatrix}.$$

Let  $H = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ . Finally, let  $\mathbf{u} = [1, 0, 0, 0]^T$ .

- (a) Find an orthogonal basis for H.
- (b) Find an orthonormal basis for H.
- (c) What is the dimension of H? (hint: it is not 4)
- (d) Find the orthogonal projection  $\mathbf{u}$  on H.
- (e) Find the distance between  $\mathbf{u}$  and H.
- (f) Find an orthonormal basis for  $H^{\perp}$ .
- (g) What is the distance between **u** and  $H^{\perp}$ ?

(h) (challenging) Explain how you could have deduced the answer to (f) from the answer in (d), and the answer to (g) from the answer to (e).