Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise $1(10=2.5+1+0.5+1.5+1+1.5+1+1$ points $)$
Let

$$
\mathbf{u}_{1}=\left[\begin{array}{l}
1 \\
1 \\
1 \\
1
\end{array}\right], \mathbf{u}_{2}=\left[\begin{array}{l}
1 \\
0 \\
0 \\
1
\end{array}\right], \mathbf{u}_{3}=\left[\begin{array}{c}
0 \\
2 \\
1 \\
-1
\end{array}\right], \mathbf{u}_{4}=\left[\begin{array}{c}
2 \\
3 \\
2 \\
1
\end{array}\right]
$$

Let $H=\operatorname{Span}\left\{\mathbf{u}_{1}, \mathbf{u}_{2}, \mathbf{u}_{3}, \mathbf{u}_{4}\right\}$. Finally, let $\mathbf{u}=[1,0,0,0]^{T}$.
(a) Find an orthogonal basis for $H$.
(b) Find an orthonormal basis for $H$.
(c) What is the dimension of $H$ ? (hint: it is not 4)
(d) Find the orthogonal projection $\mathbf{u}$ on $H$.
(e) Find the distance between $\mathbf{u}$ and $H$.
(f) Find an orthonormal basis for $H^{\perp}$.
(g) What is the distance between $\mathbf{u}$ and $H^{\perp}$ ?
(h) (challenging) Explain how you could have deduced the answer to (f) from the answer in (d), and the answer to (g) from the answer to (e).

