

120A: Homework 3

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Since we have an extra grader for the course, almost all problems will be graded in detail. The book exercises are worth 60 percent, and the extra exercises are worth 40 percent.

1. EXERCISES FROM BOOK

We will check the following exercises from the book.

Section 5: 4, 9, 22, 23, 33, 43, 47, 51

Section 6: 3, 14, 17, 23, 33, 34, 44, 48, 51

2. EXTRA EXERCISES

Exercise 1

Let $(G, *)$ and $(H, \cdot)$ be groups. Consider the set $G \times H$ of ordered pairs

$$G \times H = \{(g, h) : g \in G, h \in H\}.$$

Define an operation on $G \times H$ by

$$(g_1, h_1) * (g_2, h_2) := (g_1 * g_2, h_1 \cdot h_2).$$

- (a) Show that $(G \times H, *)$ is a group (called the *product of G and H*).
- (b) Show that $G \times H$ is abelian if and only if G and H are both abelian.
- (c) Show that $G \times \{e\} = \{(g, e) : g \in G\}$ and $\{e\} \times H = \{(e, h) : h \in H\}$ are subgroups of $G \times H$.
- (d) Give an example of non-trivial cyclic groups G, H such that $G \times H$ is not cyclic. Prove your answer.
- (e) Find all elements of order 4 in $\mathbf{Z}_4 \times \mathbf{Z}_8$.
- (f) Show that $\mathbf{Z}_3 \times \mathbf{Z}_5$ is a cyclic group and find a generator.