MATH 120A, MIDTERM

Friday, 28 October 2016

Name: ______________________________

Student ID: ___________________________

Instructions: This exam is 50 minutes. No notes, calculators, etc. are permitted. Clearly indicate your final answers, and show all work unless the problem explicitly says no justification is required. Cross out incorrect work. Do work in the space provided. Good luck.

Question	Score	Maximum
1		12
2		10
3		10
4		8
5		10
Total		50

1. (12 points) Briefly answer the following questions.

   a. Let $(\mathbb{C}, \cdot)$ denote the binary structure of complex numbers with multiplication. Is this binary structure a group? Briefly explain your answer.

   b. Among the proper subgroups of $(\mathbb{Z}_{100}, +_{100})$, identify the proper subgroup with the biggest order.

   c. If $\langle a \rangle$ is a cyclic group of order 20 generated by a, what is the inverse of a^3? Express your answer as one of the elements $\{a^0, a^1, a^2, \ldots, a^{19}\}$.

2. (10 points) Consider the element $\sigma \in S_6$ given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 3 & 2 & 1 \end{pmatrix}.$$

a. Write σ^{-1} in the same form.

b. Write σ in cycle notation.

c. Compute σ^{30}. Briefly explain your answer.

3. (10 points)

a. Prove the left cancellation law: If G is a group and $ab = ac$, then $b = c$. Be sure to indicate which properties of a group you are using.

b. Prove that if G is a group and $g, h \in G$ are elements satisfying $gh = g$, then h is the identity element of the group.

4. (8 points) Let $(G, +)$ and $(H, \cdot)$ denote isomorphic groups. Carefully prove that if G is abelian, then H is also abelian.

5. (10 points) For each of the following, either give an example or prove none exists.
   a. An infinite cyclic group.

   b. A non-abelian group G of order 24 and an element $g \in G$ of order 24.

   c. An isomorphism between the groups $(U(\mathbb{Z}_9), \cdot_9)$ and $(\mathbb{Z}_6, +_6)$.