

120A: Homework 5

By: Michiel Kusters

Report mistakes to kusters@gmail.com

Since we have an extra grader for the course, almost all problems will be graded in detail. The book exercises are worth 60 percent, and the extra exercises are worth 40 percent. Write the solutions of the two different parts on different papers.

1. EXERCISES FROM BOOK

We will check the following exercises from the book.

Section 10: 1, 4, 6, 12, 16, 19, 28, 29, 37

2. EXTRA EXERCISES

Exercise 1

Do exercises 45 and 46 from Section 10.

Exercise 2

Prove the following: A group G of order n is cyclic if and only if for every divisor d of n there are at most $\varphi(d)$ elements of order d in G (hint: use the previous exercise; φ is the Euler phi-function).

Exercise 3

Let H_1, H_2 be subgroups of a group G and assume that $H_1 \cup H_2 = G$. Show that $G = H_1$ or $G = H_2$. Does a similar statement hold for the identity $G = H_1 \cup H_2 \cup H_3$?