

120A: Homework 6

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Since we have an extra grader for the course, almost all problems will be graded in detail. The book exercises are worth 60 percent, and the extra exercises are worth 40 percent. Write the solutions of the two different parts on different papers.

1. EXERCISES FROM BOOK

We will check the following exercises from the book.

Section 11: 7, 14, 29, 36, 47, 52

Section 13: 3, 8, 10, 17, 19, 32, 49

2. EXTRA EXERCISES

Exercise 1

Let G, H, K be finite abelian groups. Show that if $G \times K$ is isomorphic to $H \times K$, then $G \cong H$.

Exercise 2

How many homomorphisms are there from $\mathbf{Z}_2 \rightarrow S_3$? How many homomorphisms are there from $\mathbf{Z}_3 \rightarrow S_3$. How many homomorphisms are there from $\mathbf{Z}_5 \rightarrow S_3$?

Exercise 3

- (a) Prove that if every non-identity element in a group has order 2, then the group is abelian.
- (b) Is $\mathbf{Z}_{250} \times \mathbf{Z}_{50}$ isomorphic to $\mathbf{Z}_{50} \times \mathbf{Z}_{25}$?
- (c) Up to isomorphism, how many abelian groups are there of order 1250.
- (d) Prove that if a group has order 1250, then G has an element of order 5 (Hint: Use Lagrange's theorem).