120A: Homework 7

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Since we have an extra grader for the course, almost all problems will be graded in detail. The book exercises are worth 60 percent, and the extra exercises are worth 40 percent. Write the solutions of the two different parts on different papers.

1. Exercises from book

We will check the following exercises from the book.

Section 14: 3, 5, 9, 12, 26, 30, 31, 37
Section 15: 4, 7, 13, 14, 21, 35, 36

2. Extra exercises

Exercise 1
Consider the group $G = \{m+n\sqrt{3} : m, n \in \mathbb{Z}\}$. Let $H = \{4m+2n\sqrt{3} : m, n \in \mathbb{Z}\}$.
(a) Show that G is a subgroup of $(\mathbb{R}, +)$.
(b) Show that H is a subgroup of G.
(c) Explain (quickly) why H is normal in G.
(d) Explicitly list the elements of G/H.
(e) Find the order of each element of G/H.
(f) Is G/H cyclic? Is it abelian?
(g) Find a familiar group which is isomorphic to G/H.
(h) Explicitly show that the map

$$\phi : G/H \to \mathbb{Z}_4 \times \mathbb{Z}_2,\ m + n\sqrt{3} + H \mapsto (\overline{m}, \overline{n}$$

is an isomorphism. Don't forget to prove that ϕ is well defined.
(i) Give an alternative proof of the existence of an isomorphism between G/H and $\mathbb{Z}_4 \times \mathbb{Z}_2$ by applying the first isomorphism theorem to

$$\psi : G = \{m + n\sqrt{3} : m, n \in \mathbb{Z}\} \to \mathbb{Z}_4 \times \mathbb{Z}_2,\ m + n\sqrt{3} \mapsto (\overline{m}, \overline{n}).$$

Note: Make sure to check that ψ is a homomorphism, and find its Kernel and Image.

Exercise 2
Let $G = \mathbf{A}_4$.
(a) Write down all elements of G in cycle notation.
(b) Find all subgroups of G.
(c) Find all normal subgroups of G.
(d) For eachnormal subgroup H of G, determine to which known group G/H is isomorphic.