120B: Homework 1

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1. Book exercises

Complete the following book exercises: Section 18: 8, 27, 33, 39, 41, 56.

2. Extra exercises

Exercise 1
We know that $(\mathbf{Z}, +, \cdot)$ is a ring. For each of the following triples, determine whether the given triple is a ring. Justify your answers.
(a) $(\mathbf{Z}, +, +)$.
(b) $(\mathbf{Z}, +, \star)$ where $\star$ is defined by $a \star b = 0$ for all $a, b \in \mathbf{Z}$.
(c) $(\mathbf{Z}, \cdot, +)$.

Exercise 2:
For each of the following, determine if it is a ring. If it is not a ring, prove it. If it is a ring, answer the following questions and prove your answers.

i. Is it commutative?
ii. Does it have unity? If so, identify it.
iii. Is it a field?

(a) The natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ with the usual operations of addition and multiplication.
(b) The "integers and half-integers" $\{\ldots, -3/2, -1, -1/2, 0, 1/2, 1, \ldots\}$, i.e., the rational numbers $r \in \mathbf{Q}$ satisfying $2r \in \mathbf{Z}$.
(c) The set is $\prod_{n=1}^{\infty} \mathbf{R}$, which is sequences of real numbers $(a_1, a_2, \ldots)$, with addition and multiplication defined termwise.
(d) The set is $\bigoplus_{n=1}^{\infty} \mathbf{R}$, which is sequences of real numbers $(a_1, a_2, \ldots)$ for which all but finitely many of the numbers are zero, and with addition and multiplication defined termwise.