

## 120B: Homework 2

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### 1. BOOK EXERCISES

Complete the following book exercises:

Section 19: 1, 10, 17, 26, 30.

Section 20: 6, 7, 17.

### 2. EXTRA EXERCISES

#### Exercise 1

For each of the following, determine if it is a ring. If it is not a ring, prove it. If it is a ring, answer the following questions and prove your answers.

- i. Is it commutative?
  - ii. Does it have unity? If so, identify it.
  - iii. Is it an integral domain?
  - iv. Is it a field?
- (a) The set of all rational numbers that can be written in the form  $\frac{m}{n}$  with  $m, n \in \mathbf{Z}$  and  $n$  odd, with the usual operations of addition and multiplication.
- (b) The set of all functions  $f : \mathbf{R} \rightarrow \mathbf{R}$ , with operations of pointwise addition and multiplication of functions.
- (c) The set of all  $2 \times 2$  matrices with real entries and trace equal to zero, with the usual matrix addition and matrix multiplication.
- (d) The set of all subsets of  $\mathbf{Z}$  with operations defined by  $S_1 + S_2 = S_1 \cup S_2$  and  $S_1 \cdot S_2 = S_1 \cap S_2$ .
- (e) The set  $\mathbf{Q} \times \mathbf{Q}$  with operations defined by  $(a, b) + (c, d) = (a + c, b + d)$  and  $(a, b) \cdot (c, d) = (ac - bd, ad + bc)$ .

#### Exercise 2

Compute the last two digits of  $(3^4)^5$  and  $3^{4^5}$ .