Math 120B: Midterm

February 17, 2017

Last Name: ______________________________

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Student ID: ______________________________

Instructions: This exam is 50 minutes. No notes, calculators, etc. are permitted. Clearly indicate your final answers, and show all work unless the problem explicitly says no justification is required. Cross out incorrect work. Do work in the space provided. Good luck.

1. (12 points) The following tables describe a ring R; use the tables to answer the following questions.

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	c	a	c
d	a	d	c	b

a. Which elements in this ring are units?

b. Is this ring an integral domain? Explain your answer.

c. Is this ring isomorphic to $\mathbb{Z}_4$?

2. (8 points)

   a. Prove that no element in a field can be a zero divisor. (Recall that, by definition, 0 is not a zero divisor.)

   b. Give an example of elements a, b in a ring R such that $ab = 0$ but $ba \neq 0$.

3. (15 points) Answer True or False to each of the following, and briefly justify your answers.

a. True/False: $\mathbb{R}[x]$ is isomorphic to $\mathbb{R} \times \mathbb{R}$.

b. True/False: The map $\mathbb{Z}_5 \to \mathbb{Z}_{10}$ given by $a \mapsto 6a \bmod 10$ is a ring homomorphism.

c. True/False: If R is a field and $r \in R$ is non-zero, then there exists an integer $k \geq 1$ such that $r^k = 1$.

4. (15 points) For each of the following, give an example or briefly explain why no example exists.

   a. An integral domain which is not a field.

   b. A commutative ring with unity R and a degree 2 polynomial $f(x) \in R[x]$ such that $f(x)$ has at least three zeros in R.

   c. A polynomial $f(x) \in \mathbb{Q}[x]$ such that $f(x)$ is irreducible over $\mathbb{Q}$ but is reducible over $\mathbb{R}$.