120B: Homework 3
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1. Book exercises

Complete the following book exercises:
Section 20: 29.
Section 21: 1, 4.
Section 22: 1, 17, 23, 27, 30.

2. Extra exercises

Exercise 1
Let F be a finite field. Let $\varphi : F \to F$ be any function. Show that there is a polynomial $f \in F[X]$ such that for any $c \in F$ one has $f(c) = \varphi(c)$ (hint: for every $d \in F$ construct a polynomial $g_d \in F[X]$ with $g_d(c) = \delta_{cd}$ for any $c \in F$).

Exercise 2
Let $m, n \in \mathbf{Z}_{\geq 1}$ with $\gcd(m, n) = 1$.
(a) Show that the natural map

$$\begin{aligned} \tau : \mathbf{Z}/mn\mathbf{Z} &\to \mathbf{Z}/m\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z} \\ a + mn\mathbf{Z} &\mapsto (a + m\mathbf{Z}, a + n\mathbf{Z}) \end{aligned}$$

is an isomorphism of rings (prove first that the map is well-defined). You may use that by the Euclidean algorithm, there are $x, y \in \mathbf{Z}$ with $xm + yn = 1$.
(b) Show that $\varphi(mn) = \varphi(m)\varphi(n)$ (here φ is the Euler phi function, defined by $\varphi(m) = \#(\mathbf{Z}/m\mathbf{Z})^*$).