

120B: Homework 6

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1. BOOK EXERCISES

Complete the following book exercises:

Section 29: 2, 8, 10, 12, 16, 23, 30, 32, 33.

2. EXTRA EXERCISES

Exercise 1

Let $K = \mathbf{Q}(\alpha)$ with $\text{irr}(\alpha, \mathbf{Q}) = X^3 + 2X^2 + 1$. Recall that any element in K can be expressed uniquely as $a_0 + a_1\alpha + a_2\alpha^2$ with $a_0, a_1, a_2 \in \mathbf{Q}$. Hint for the whole exercise: linear algebra!

- (a) Compute the inverse of $\alpha + 1$ (in the form $a_0 + a_1\alpha + a_2\alpha^2$).
- (b) Compute $\text{irr}(\alpha^2, \mathbf{Q}) \in \mathbf{Q}[X]$.

Exercise 2

Let F be a field such that every $f \in F[x]$ of degree at least 1 has a zero in F . Show that F is infinite.