120B: Homework 7

By: Michiel Kosters
Report mistakes to kosters@gmail.com

---

1. Book exercises

Complete the following book exercises:
Section 30: 9, 10, 15, 23
Section 31: 1, 2, 6, 19, 23, 24, 30, 31, 36.

2. Extra exercises

Exercise 1
Let L/K be a finite algebraic extension. Prove that for $\alpha, \beta \in L$:

$$[K(\alpha, \beta) : K] \leq [K(\alpha) : K] \cdot [K(\beta) : K].$$

Show that this inequality is not always an equality. Is there equality if $[K(\alpha) : K]$ and $[K(\beta) : K]$ are coprime?

Exercise 2
Let $\mathbf{C}(X)$ be the field of rational functions with complex coefficients. Show that a $\mathbf{C}$-basis of $\mathbf{C}(X)$ is given by

$$\left\{X^i\right\}_{i=0}^{\infty} \bigcup \left\{ \frac{1}{(X-\alpha)^k} : \alpha \in \mathbf{C}, k \in \mathbf{Z}_{>0} \right\}.$$

(This problem is related to the partial fraction methods you learned in Math 2B).