Math 2B: Quiz 2B Solutions

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Exercise 1 (3 points) Find the derivative of

$$g(x) = \int_{10x}^{x^2} \pi^t \, dt$$

Solution:

$$g(x) = \int_{10x}^{0} \pi^t dt + \int_{0}^{x^2} \pi^t dt = \int_{0}^{x^2} \pi^t dt - \int_{0}^{10x} \pi^t dt$$

Using FTC and Chain Rule, we get

$$g'(x) = 2x\pi^{x^2} - 10\pi^{10x}$$

Exercise 2 (3 points) Evaluate

$$\int_1^2 \left(\sqrt{\frac{8}{z}} + z^e \right) dz$$

Solution:

$$\int_1^2 \sqrt{\frac{8}{z}} dz + \int_1^2 z^e dz = \sqrt{8} \int_1^2 z^{-\frac{1}{2}} dz + \int_1^2 z^e dz$$

$$= (2\sqrt{8} z^{\frac{1}{2}} + \frac{z^{e+1}}{e+1})|_{z=1}^{z=2} = (2\sqrt{8})2^{\frac{1}{2}} + \frac{2^{e+1}}{e+1} - (2\sqrt{8})1^{\frac{1}{2}} - \frac{1^{e+1}}{e+1}$$

Exercise 3 (4 points)
(a) Use the Fundamental Theorem of Calculus to evaluate

$$\int_2^4 3x \, dx$$

(b) Express the integral from part (a) as a limit of Riemann sums.
(c) Use the limit of Riemann sums definition to evaluate the integral in part (a). You will need $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.

Solution:

(a)

$$\frac{3}{2}x^2|_{x=2}^{x=4} = 24 - 6 = 18$$

(b)

$$f(x) = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{2}{n}, x_i = 2 + i\frac{2}{n}$$

(c)

$$f(x) = \lim_{n\to\infty} \sum_{i=1}^{n} 3(2 + i\frac{2}{n})\frac{2}{n} = \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n}(6 + 6i\frac{2}{n}) = \lim_{n\to\infty} (\frac{2}{n}) \sum_{i=1}^{n} 6 + \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} 6i\frac{2}{n}$$

$$= \lim_{n\to\infty} (\frac{12}{n}) \sum_{i=1}^{n} 1 + \lim_{n\to\infty} \frac{12}{n^2} \sum_{i=1}^{n} i = \lim_{n\to\infty} (\frac{12}{n})(n) + \lim_{n\to\infty} \frac{12}{n^2} \left(\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} 12 + \lim_{n\to\infty} \frac{12n^2 + 12n}{2n^2} = 12 + 6 = 18$$