Math 2B: Quiz 3B Solutions

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Exercise 1(4 points) Evaluate the integral

$$\int_0^{\pi^{\frac{1}{3}}} 2x^2 \sin(x^3)dx$$

Let $u = x^3$. Then $du = 3x^2dx$. So

$$\int_0^{\pi^{\frac{1}{3}}} 2x^2 \sin(x^3)dx = \frac{2}{3}\int_0^{\pi^{\frac{1}{3}}} 3x^2 \sin(x^3)dx = \frac{2}{3}\int_0^{\pi} \sin(u)du = \frac{2}{3}[-cosu]|_{u=0}^{u=\pi} = \frac{4}{3}$$

Exercise 2 (6 points) Find the volume of the three dimensional solid obtained by rotating the region bounded by the given curves about the specified line.

(a) $y = x^2, y = x, x \geq 0$; about the x-axis

$$\int_0^1 \pi(x^2 - x^4)dx = \pi(\frac{x^3}{3} - \frac{x^5}{5})|_{x=0}^{x=1} = \frac{\pi}{3} - \frac{\pi}{5}$$

(b) $2x = y^2, x = 0, y = 4$; about the y-axis

$$\int_0^4 \pi(\frac{y^2}{2})^2 dy = \int_0^4 \pi(\frac{y^4}{4})dy = \pi(\frac{y^5}{5})|_{y=0}^{y=4} = \pi\frac{4^5}{5}$$