Math 2B: midterm 2

Wednesday November 15 2017, 8:00 – 8.50am

There are 5 exercises, worth a total of 93 points.
No electronic devices/books/notes allowed.
Provide computations and or explanations, unless stated otherwise.

Name:

Student ID:

Exercise 1 (36 = 9 + 9 + 9 + 9 pts) Evaluate each of the following integrals.

(a) $\int e^x \cos(x)\,\mathrm{d}x$

(b) $\int \frac{\sqrt{x^2-1}}{x^4}\,\mathrm{d}x$

(c) $\int \cos^5(\theta)\sin^2(\theta)\,\mathrm{d}\theta$

(d) $\int \frac{\sqrt{x-2}}{x}\,\mathrm{d}x$

Exercise 2 (24 = 8 + 8 + 8 pts) Determine whether the following improper integrals are convergent or divergent (hint: first compute the indefinite integrals). Evaluate those that are convergent.

(a) $\int_0^3 \frac{\mathrm{d}x}{x}$.

(b) $\int_1^\infty \frac{\mathrm{d}x}{x^2\sqrt{x^2+4}}$

(c) $\int_0^\infty \frac{\mathrm{d}y}{(y+1)(y+2)}$

Exercise 3 (15 = 3 + 3 + 3 + 3 + 3 pts) Determine whether each of the following sequences $\{a_n\}_{n=1}^{\infty}$ is convergent or divergent. If a sequence is convergent, find its limit.

(a) $a_n = 1 + \frac{(-1)^n}{n^2}$

(b) $a_n = \cos\left(\frac{\pi n^5 + n^3}{n^5 + 4n^2}\right)$

(c) $a_n = \sin(\frac{\pi}{2} n)$

(d) $a_n = (\frac{4}{5})^n$

(e) $a_n = n \ln(1 + 1/n)$.

Exercise 4 (10 = 5 + 5 pts)
(a) Compute $\sum_{n=0}^{\infty} \frac{5 \cdot 3^{n+1}}{2^{2n}}$.

(b) Compute $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$.

Exercise 5 (8 = 2 + 2 + 2 + 2 pts) Determine whether each of the following statements is true or false. No justification needed.
(a) Suppose f and g are continuous with $f(x) \geq g(x)$ for $x \geq a$. If $\int_a^{\infty} f(x)\,\mathrm{d}x$ is convergent, then $\int_a^{\infty} g(x)\,\mathrm{d}x$ is convergent.

(b) The sum $\sum_{n=1}^{\infty} \frac{n^2+3n}{n^2+2}$ converges.

(c) $\int \sec(x)\,\mathrm{d}x = \ln|\cos(x) + \tan(x)| + C$.

(d) Every bounded, increasing sequence is convergent.

Solutions:
1a: (integration by parts) $e^x(\sin(x)+\cos(x))/2+C$.
1b: $(x^2-1)^{3/2}/(3x^3)+C$
1c: $1/7\sin^7(\theta)-2/5\sin^5(\theta)+1/3\sin^3(\theta)+C$.
1d: $2\sqrt{x-2}-2\sqrt{2}\tan^{-1}(\sqrt{x-2}/\sqrt{2})$.
2a: $\lim_{t\to 0}-\ln(t)=\infty$. diverges
2b: antiderivative $-\frac{\sqrt{x^2+4}}{4x}+C$, integral becomes $1/4(\sqrt{5}-1)$.
2c: $-1/(y+1)-1/3\cdot 1/(y+2)$, integral becomes $\ln(2)$.
3a: 1.
3b: $\cos(\pi)=-1$
3c: diverges
3d: 0
3e: 1 (use l'Hôpital)
4a: $\sum_{n=0}^{\infty}15(3/4)^n=15/(1-3/4)=60$
4b: (telescoping): diverges
5a: false (functions must be non-negative)
5b: false (terms do not go to 0)
5c: false (sec instead of cos)
5d: true