Math 2B: Quiz 7 Solutions

Exercise 1 (10 points)Determine whether the series below converge or diverge. Justify your answer. (You may use the back page)
(a)

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

$$\int_2^{\infty} \frac{1}{x\ln(x)}dx = lim_{h\to\infty} \int_2^{h} \frac{1}{x\ln(x)}dx = lim_{h\to\infty} \int_{\ln(2)}^{\ln(h)} \frac{1}{u}du = \ln(\ln(h)) - \ln(\ln(2)) = \infty$$

so integral diverges by integral test
(b)

$$\sum_{n=1}^{\infty} \frac{1}{5^n+2}$$

$$\sum_{n=1}^{\infty} \frac{1}{5^n+2} \le \sum_{n=1}^{\infty} \frac{1}{5^n}$$

which is a convergent geometric series so we have our sum converges by comparison test.
(c)

$$\sum_{n=1}^{\infty} \frac{1}{n^6+10}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^6+10} \le \sum_{n=1}^{\infty} \frac{1}{n^6}$$

which is a convergent p series so we have our sum converges by comparison test.
(d)

$$\sum_{n=1}^{\infty} \frac{1}{5^n-1}$$

Note that $5^{n-1} \le 5^n - 1$ so $\frac{1}{5^{n-1}} \ge \frac{1}{5^n-1}$. So

$$\sum_{n=1}^{\infty} \frac{1}{5^n-1} \le \sum_{n=1}^{\infty} \frac{1}{5^{n-1}}$$

which is a convergent geometric series so we have our series converges by comparison test.
(e)

$$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$$

$\ln(n) \leq n$ so $\frac{1}{\ln(n)} \geq \frac{1}{n}$ so

$$\sum_{n=2}^{\infty} \frac{1}{n} \leq \sum_{n=2}^{\infty} \frac{1}{\ln(n)}$$

As the harmonic series diverges, our sum diverges by comparison test.