Math 2B: Quiz 8 Solutions

Exercise 1 (10 points)Determine whether the series below converge or diverge. Justify your answer. (You may use the back page)

(a)

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{\frac{1}{2}}}$$

converges by alternating series test

(b)

$$\sum_{n=2}^{\infty} \left(\frac{n}{\ln(n)}\right)^n$$

$$lim_{n\to\infty} \frac{n}{\ln(n)} = \infty > 1$$

So we have that this series diverges by root test

(c)

$$\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^2}$$

$$lim_{n\to\infty} \frac{(3n+3)!}{(n+1)!(n+1)!} \frac{n!n!}{(3n)!}$$

$$= lim_{n\to\infty} \frac{(3n+3)(3n+2)(3n+1)}{(n+1)(n+1)} = \infty > 1$$

So this series diverges by ratio test

(d)

$$\sum_{n=1}^{\infty} \frac{3^n}{(n!)2^n}$$

$$\lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!2^{n+1}} \frac{n!2^n}{3^n}$$

$$= \lim_{n\to\infty} \frac{3}{2(n+1)} = 0 < 1$$

So the series converges by ratio test

(e)

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$$

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e > 1$$

So the series diverges by root test

(f)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{1}{3}}}$$

converges by alternating series test