CS 171, Intro to AI — Quiz #2 — Fall Quarter, 2017 — 20 minutes

YOUR NAME: _____

YOUR ID: _____

Many students asked about partial credit for this problem. If your answer is mathematically correct, is responsive to the problem, and showed that you knew what you were doing and had mastered the material, then you get full credit even if it is not in one of the forms below. In general, if your answer showed that you knew what you were doing and had mastered the material but had a very minor mistake, then you are likely to get partial credit. If your answer did not show that you knew what you were doing and had mastered the material, then you are likely to get little or no credit. The Readers will use their own best judgement, and their decision is final.

1. (42 pts total, 6 pts each) PROBABILITY FORMULAS. Write out the following probability formulas. Below, "in terms of X" means X should appear in your answer. All answers should be formulas, not text.

1.a. (6 pts) Write the formula for P(A ∧ B) in terms of P(A ∨ B) and possibly other terms.

P(A ∧ B) = P(A) + P(B) - P(A ∨ B)

Other answers get full credit if they are mathematically correct. E.g., P(A ∧ B) = P(A ∨ B) - P(A ∧ ¬B) - P(B ∧ ¬A) is creative, but it gets full credit because it is mathematically correct & responsive to the problem.

1.b. (6 pts) Write the formula for the conditional probability P(A | B).

P(A | B) = P(A ∧ B) / P(B)

1.c. (6 pts) Factor P(A ∧ B ∧ C) completely using the Product Rule (or Chain Rule). You may use any variable ordering you wish.

P(A ∧ B ∧ C) = P(A | B ∧ C) * P(B | C) * P(C)

Other variable orderings are OK iff correct, e.g.,
P(A ∧ B ∧ C) = P(C | A ∧ B) * P(B | A) * P(A)
= P(B | A ∧ C) * P(C | A) * P(A), etc.

1.d. (6 pts) Given a joint probability distribution P(A ∧ B ∧ C), use the Sum Rule (or Law of Total Probability) to write the marginal probability of P(A).

P(A) = $\Sigma_{B, C}$ P(A ∧ B ∧ C)

All are correct:
P(A) = $\Sigma_B \Sigma_C$ P(A ∧ B ∧ C)
= $\Sigma_{b \in B} \Sigma_{c \in C}$ P(A ∧ b ∧ c)
= $\Sigma_{b \in B, c \in C}$ P(A ∧ b ∧ c)

1.e. (6 pts) Write Bayes' Rule (or Bayes' Theorem).

P(A | B) = P(B | A) * P(A) / P(B) = $\frac{P(A \mid B) * P(A)}{\Sigma_{a \in A} P(B \mid a) * P(a)}$ = $\frac{P(A \mid B) * P(A)}{P(B \mid a) * P(a) + P(B \mid \neg a) * P(\neg a)}$

1.f. (6 pts) Assume that A and B are independent. Write P(A ∧ B) in terms of P(A) and P(B) and possibly other terms.

P(A ∧ B) = P(A) * P(B)

Bayes' Rule is written in several different forms in different places, any of which gets full credit if mathematically correct.

Iff A is Boolean.

1.g. (6 pts) Assume that A and B are conditionally independent given C. Write P(A ∧ B | C) in terms of P(A | C) and P(B | C) and possibly other terms.

P(A ∧ B | C) = P(A | C) * P(B | C)

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Question 2 is Exercise 8.9(i-iii), R&N, 3rd ed., p. 316.
A minor typo has been corrected: Border = Borders.

2. (40 points total, 5 pts each) English to FOL Sentence write **C** if the FOL sentence Correctly expresses the English sentence, write **I** if it is syntactically invalid and therefore meaningless; or **N** if it it is syntactically valid but does Not correctly express the English sentence. Note that $\wedge$ is a stronger operator than is $\Rightarrow$. **The first one is done for you as an example.**

2.Example. "Paris and Marseilles are both in France."

A. (Example) ___I___ (Write C or I or N) *In(Paris $\wedge$ Marseilles, France)*
Paris $\wedge$ Marseilles is a conjunction of terms, not a conjunction of truth values, and so is syntactically Invalid.
B. (Example) ___C___ (Write C or I or N) *In(Paris, France) $\wedge$ In(Marseilles, France)*
Correct. Paris is in France, AND Marseilles is in France,
C. (Example) ___N___ (Write C or I or N) *In(Paris, France) $\vee$ In(Marseilles, France)*
This sentence says that Paris is in France OR Marseilles is in France, a disjunction instead of a conjuction.

2.a. (20 pts total, 5 pts each) "There is a country that borders both Iraq and Pakistan."

A. (5 pts) ___C___ (Write C or I or N) $\exists c$ *Country(c) $\wedge$ Borders(c, Iraq) $\wedge$ Borders(c, Pakistan)*
Correct. There exists a Country c, AND c Borders Iraq, AND c Borders Pakistan.
B. (5 pts) ___N___ (Write C or I or N) $\exists c$ *Country(c) $\Rightarrow$ [Borders(c, Iraq) $\wedge$ Borders(c, Pakistan)]*
This sentence is vacuously true if there exists anything that is not a Country, for example, the lecture podium.
(For non-native-English speakers: vacuously comes from the same root as vacuum, which means empty.
Thus, a statement is vacuously true if it is true, but it is empty of content or meaning.)
C. (5 pts) ___I___ (Write C or I or N) *[$\exists c$ Country(c)] $\Rightarrow$ [Borders(c, Iraq) $\wedge$ Borders(c, Pakistan)]*
This sentence is syntactically invalid because the scope of $\exists c$ does not extend to the consequent.Remember that FOL does not allow free variables, so the consequent is not syntactically well formed.
D. (5 pts) ___I___ (Write C or I or N) $\exists c$ *Borders(Country(c), Iraq $\wedge$ Pakistan)*
This sentence is syntactically invalid because "Iraq $\wedge$ Pakistan" is not a syntactically valid term.

2.b. (20 pts total, 5 pts each) "All countries that border Ecuador are in South America."

A. (5 pts) ___C___ (Write C or I or N) $\forall c$ *Country(c) $\wedge$ Borders(c, Ecuador) $\Rightarrow$ In(c, SouthAmerica)*
Correct. Anything that is a Country and Borders Ecuador is in South America.
B. (5 pts) ___C___ (Write C or I or N) $\forall c$ *Country(c) $\Rightarrow$ [Borders(c, Ecuador) $\Rightarrow$ In(c, SouthAmerica)]*
Correct. If c is a Country, then if c Borders Ecuador, then c must be in SouthAmerica.
This sentence is just another way of rewriting sentence A above, using (A $\Rightarrow$ B) = ($\neg$A $\vee$ B).
C. (5 pts) ___N___ (Write C or I or N) $\forall c$ *[Country(c) $\Rightarrow$ Borders(c, Ecuador)] $\Rightarrow$ In(c, SouthAmerica)*
This sentence is false for anything that is not a Country (so the first implication is true, and so the antecedent of the second implication is true, forcing the consequent of the second implication to be true) and also is not In SouthAmerica (so the second implication is false). E.g., for c = the lecture podium $\neg$Country(c), so the first implication is true,forcing In(c, SouthAmerica) to be true and forcing the lecture podium to be in SouthAmerica.
D. (5 pts) ___N___ (Write C or I or N) $\forall c$ *Country(c) $\wedge$ Borders(c, Ecuador) $\wedge$ In(c, SouthAmerica)*
This sentence says that everything is a Country and Borders Ecuador and is In SouthAmerica.

3. (18 pts total, 3 pts each) LOGIC CONCEPTS. For each of the following terms on the left, write in the letter corresponding to the best answer or definition on the right. **The first one is done for you as an example.**

. **A** .	Agent	. A .	Perceives environment by sensors, acts by actuators.
C	Unsatisfiable	B	Describes a sentence that is true in all models.
D	Sound	C	Describes a sentence that is false in all models.
G	Complete	D	An inference procedure that derives only entailed sentences.
B	Valid	E	Describes a sentence that is true in some model.
E	Satisfiable	F	The idea that a sentence follows logically from other sentences.
F	Entailment	G	An inference procedure that derives all entailed sentences.