Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1

Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 3 & 6 & 0 & 3 & 1 \\ 2 & 4 & 2 & 1 & 1 \end{bmatrix}$ and vector $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Finally, consider the vector $\mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 3 \end{bmatrix}.$

a) Compute Ac.

b) Find the solution(s) of the equation

 $A\mathbf{x} = \mathbf{b}$

in parametric vector form.

c) Find the solution(s) of the equation

$$A\mathbf{x} = \mathbf{0}$$

in parametric vector form.

d) Determine whether the first, second and fourth columns of A are linearly independent or not.

e) Let $T : \mathbf{R}^5 \to \mathbf{R}^3$ be the linear transformation whose standard matrix is A. Is T one-to-one? Is T onto? Recall: onto if and only if reduced row echelon form of corresponding matrix A has no zero row, and one-to-one if every column of A has a pivot.