

3A: Extra exercises 4

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 (4 points)

- (a) Let A be an $n \times n$ matrix which is invertible. Prove that $A^T A$ is invertible. Be sure to justify each step in your proof completely.
- (b) Find a 2×2 matrix A with $\text{Nul}(A) = \text{Col}(A)$. Does such an example exist when A is a 3×3 matrix?

Exercise 2 (6 points)

Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

- (a) Compute the reduced row echelon form of A . (2 points)
- (b) Find a basis of the null space of A . (1 point)
- (c) What is the dimension of the null space of A ? (1/2 point)
- (d) Find a basis of the column space of A . (1 point)
- (e) What is the rank of A ? (1/2 point)
- (f) Find all possible subsets of the columns of A which form a basis of the column space of A (tricky, 1 point).