3A: Extra exercises 6

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1
Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear map given by the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \\ 1 & 1 & 1 \end{bmatrix}.$$

Consider the basis

$$\mathcal{B} = \{[1,1,0]^T, [1,-1,1]^T, [0,2,-2]^T\}$$

of $\mathbf{R}^3$. Compute $[T]_{\mathcal{B}} = [T]_{\mathcal{B}}^{\mathcal{B}}$.

Exercise 2
(This exercise requires you to read the first half of section 5.5).
Let $\theta \in [0, 2\pi]$. Consider the (rotation) matrix

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

over the complex numbers $\mathbf{C}$.
(a) Compute the eigenvalues of A.
(b) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$ over the complex numbers.