$\qquad$
YOUR ID: $\qquad$ ID TO RIGHT: $\qquad$ ROW: $\qquad$ SEAT: $\qquad$

1. ( $\mathbf{3 0} \mathbf{~ p t s ~ t o t a l , ~} \mathbf{5} \mathbf{~ p t s ~ e a c h ) ~ R E S O L U T I O N . ~ A p p l y ~ r e s o l u t i o n ~ t o ~ e a c h ~ o f ~ t h e ~ f o l l o w i n g ~ p a i r s ~ o f ~ c l a u s e s , ~ t h e n ~}$ simplify. Write your answer in Conjunctive Normal Form (CNF), or write "None" if no resolution is possible.
1.a. (5 pts) ( $\mathrm{P} Q \neg \mathrm{R} \mathrm{S}$ ) ( $\mathrm{P} \neg \mathrm{Q} \mathrm{W} \mathrm{X}$ ). $\qquad$ .
1.b. (5 pts) ( $\mathrm{P} Q \neg \mathrm{R} \mathrm{S}$ ) ( $\neg \mathrm{P}$ ). $\qquad$
1.c. (5 pts) ( $\neg \mathrm{R}$ ) (R). $\qquad$ .
1.d. (5 pts) ( $\mathrm{P} \mathrm{Q} \neg \mathrm{R} \mathrm{S}$ ) ( $\mathrm{P} \mathrm{R} \neg \mathrm{S} \mathrm{W} \mathrm{X}$ ). $\qquad$ .
1.e. (5 pts) ( $\mathrm{P} \neg \mathrm{Q} \mathrm{R} \neg \mathrm{S}$ ) ( $\mathrm{P} \neg \mathrm{Q} \mathrm{R} \neg \mathrm{S}$ ) $\qquad$
1.f. (5 pts) ( $\mathrm{P} \neg \mathrm{Q} \neg \mathrm{S} \mathrm{W}$ ) ( $\mathrm{P} \mathrm{R} \neg \mathrm{S} \mathrm{X}$ ) $\qquad$
2. ( 5 pts each, 30 pts total) LOGIC TERMINOLOGY. In the following, KB is a set of sentences, $\}$ is the empty set of sentences, and S is a single sentence. Recall that $\mid=$ is read "entails" and that $\mid-$ is read "derives."

$$
\begin{array}{ll}
\mathbf{S}=\text { Sound. } & \mathbf{U}=\text { Unsound. } \\
\mathbf{C}=\text { Complete. } & \mathbf{I}=\text { Incomplete. } \\
\text { Sat = Satisfiable. } & \text { Unsat = Unsatisfiable. } \\
\mathbf{V}=\text { Valid. } & \mathbf{N}=\text { None of the above. }
\end{array}
$$

For each blank below, write in the key above that corresponds to the best term.
2.a. Let $S$ be given in advance. Suppose that $\} \mid=S$. Then $S$ is $\qquad$ .
2.b. Let S be given in advance. Suppose that for some $\mathrm{KB} 1, \mathrm{KB1} \mid=\mathrm{S}$; but that for some other KB2, KB2 $\mid=\neg \mathrm{S}$. Then $S$ is $\qquad$
2.c. Suppose that for any KB and any S , whenever $\mathrm{KB} \mid=\mathrm{S}$ then $\mathrm{KB} \mid-\mathrm{S}$.

Then the inference procedure is $\qquad$ .
2.d. Suppose that for some $K B$ and some $S, K B \mid-S$ but not $K B \mid=S$.

Then the inference procedure is $\qquad$ .
2.e. Suppose that for some KB and some $\mathrm{S}, \mathrm{KB} \mid=\mathrm{S}$ but not $\mathrm{KB} \mid-\mathrm{S}$.

Then the inference procedure is $\qquad$ ..
2.f. Suppose that for any KB and any S , whenever $\mathrm{KB} \mid-\mathrm{S}$ then $\mathrm{KB} \mid=\mathrm{S}$. Then the inference procedure is $\qquad$ .

Amy, Betty, Cindy, and Diane went out to lunch at a seafood restaurant. Each ordered one fish. Each fish was either a red fish or a blue fish. Among them they had exactly three red fish and one blue fish.

You translate this fact into Propositional Logic (in prefix form) as:
/* Ontology: Symbol A/B/C/D means that Amy/Betty/Cindy/Diane had a red fish. */
$\begin{array}{lll}\text { (or } & \text { (and } A B C(\neg D) & \text { (and } A B(\neg C) D) \\ & \text { (and } A(\neg B) C D) & \text { (and }(\neg A) B C D) \text { ) }\end{array}$
Their waiter reported:
"Amy and Cindy had the same color fish; I don't remember which color it was.
Cindy and Diane had the same color fish; I don't remember which color it was."
You translate these facts into Propositional Logic (in prefix form) as:
(<=> A C) (<=> C D)
Betty's daughter asked, "Is it true that my mother had a blue fish?"
You translate this query into Propositional Logic as " $(\neg \mathrm{B})$ " and form the negated goal as "(B)".
Your resulting knowledge base (KB) plus the negated goal (in CNF clausal form) is:

| $(A B)$ | $(A C)$ | $(A D)$ | $(B C)$ | $(B D)$ | $(C D)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $((\neg A)(\neg B)(\neg C)(\neg D))$ |  |  |  |  |  |
| $((\neg A) C)$ | $(A(\neg C))$ | $((\neg C) D)$ | $(C(\neg D))$ |  |  |
| $(B)$ |  |  |  |  |  |

(B)

## Write a resolution proof that Betty had a blue fish.

For each step of the proof, fill in the first two blanks with CNF sentences from KB that will resolve to produce the CNF result that you write in the third (resolvent) blank. The resolvent is the result of resolving the first two sentences. Add the resolvent to KB, and repeat. Use as many steps as necessary, ending with the empty clause. The empty clause indicates a contradiction, and therefore that KB entails the original goal sentence.

The shortest proof I know of is only five lines long. (A Bonus Point is offered for a shorter proof.) Longer proofs are OK provided they are correct. Obviously, it must be that Amy, Cindy, and Diane had the three red fish, so Betty must have had a blue fish. Think about it, then find a proof that mirrors how you think.

Resolve $\qquad$ with $\qquad$ to produce: $\qquad$
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