## CS-171, Intro to A.I. — Quiz#3 — Winter Quarter, 2018 — 25 minutes

YOUR NAME AND EMAIL ADDRESS: ______________________________________

YOUR ID: ____________ ID TO RIGHT: ____________ ROW: _________ SEAT: _________

**1. (30 pts total, 5 pts each) RESOLUTION.** Apply resolution to each of the following pairs of clauses, then simplify. Write your answer in Conjunctive Normal Form (CNF), or write "None" if no resolution is possible.

**1.a. (5 pts)** (P Q ¬R S) (P ¬Q W X). ______(P ¬R S W X)______ See Section 7.5.2 and Figure 7.13

**1.b. (5 pts)** (P Q ¬R S) (¬P). ______(Q ¬R S)__________________ Order of literals within clauses does not matter.

**1.c. (5 pts)** (¬R) (R). _______( )_____"FALSE" is OK__________________________.

**1.d. (5 pts)** (P Q ¬R S) (P R ¬S W X). _____(P Q ¬R R W X) also OK (P Q S ¬S W X)__.
"TRUE" is OK

**1.e. (5 pts)** (P ¬Q R ¬S) (P ¬Q R ¬S) ____________None______________________________

**1.f. (5 pts)** (P ¬Q ¬S W) (P R ¬S X) ______None______________________________

**2. (5 pts each, 30 pts total) LOGIC TERMINOLOGY.** In each of the following, KB is a set of sentences, {} is the empty set of sentences, and S is a single sentence. Recall that |= is read "entails" and that |- is read "derives."

| | |
|---|---|
| **S** = Sound. | **U** = Unsound. |
| **C** = Complete. | **I** = Incomplete. |
| **Sat** = Satisfiable. | **Unsat** = Unsatisfiable. |
| **V** = Valid. | **N** = None of the above. |

For each blank below, write in the key above that corresponds to the best term.

**2.a.** Let S be given in advance. Suppose that {} |= S. Then S is ___**V**_____ .

**2.b.** Let S be given in advance. Suppose that for some KB1, KB1 |= S; but that for some other KB2, KB2 |= ¬S. Then S is __________**Sat**_____.

**2.c.** Suppose that for any KB and any S, whenever KB |= S then KB |- S.
Then the inference procedure is __________**C**_____.

**2.d.** Suppose that for some KB and some S, KB |- S but not KB |= S.
Then the inference procedure is __________**U**_____.

**2.e.** Suppose that for some KB and some S, KB |= S but not KB |- S.
Then the inference procedure is __________**I**___..

**2.f.** Suppose that for any KB and any S, whenever KB |- S then KB |= S.
Then the inference procedure is __________**S**_____ .

****** TURN PAGE OVER. QUIZ CONTINUES ON THE REVERSE. ******

**3. (40 pts total) ONE FISH, TWO FISH, RED FISH, BLUE FISH. (With apologies to Dr. Seuss.)**
Amy, Betty, Cindy, and Diane went out to lunch at a seafood restaurant. Each ordered one fish. Each fish was either a red fish or a blue fish. **Among them they had exactly <u>three red fish</u> and <u>one blue fish</u>.**

You translate this fact into Propositional Logic (in prefix form) as:

<u>/* Ontology: Symbol A/B/C/D means that Amy/Betty/Cindy/Diane had a red fish. */</u>

(or (and A B C (¬ D)) (and A B (¬ C) D)
(and A (¬ B) C D) (and (¬ A) B C D))

See R&N Section 7.5.2.

Their waiter reported:

**"Amy and Cindy had the same color fish; I don't remember which color it was.**
**Cindy and Diane had the same color fish; I don't remember which color it was."**

You translate these facts into Propositional Logic (in prefix form) as:

(<=> A C) (<=> C D)

**Betty's daughter asked, "Is it true that my mother had a blue fish?"**

You translate this query into Propositional Logic as "(¬ B)" and form the negated goal as "(B)".

<u>Your resulting knowledge base (KB) plus the negated goal (in CNF clausal form) is:</u>

(A B) (A C) (A D) (B C) (B D) (C D) "Three red fish."
( (¬ A) (¬ B) (¬ C) (¬ D) ) "One blue fish."
( (¬ A) C) (A (¬ C) ) ( (¬ C) D) (C (¬ D) ) A <=> C <=> D.
(B) Negated goal.

**<u>Write a resolution proof that Betty had a blue fish.</u>**

For each step of the proof, fill in the first two blanks with CNF sentences from KB that will resolve to produce the CNF result that you write in the third (resolvent) blank. The resolvent is the result of resolving the first two sentences. Add the resolvent to KB, and repeat. Use as many steps as necessary, ending with the empty clause. The empty clause indicates a contradiction, and therefore that KB entails the original goal sentence.

The shortest proof I know of is only five lines long. **<u>(A Bonus Point is offered for a shorter proof.)</u>** Longer proofs are OK provided they are correct. *<u>Obviously, it must be that Amy, Cindy, and Diane had the three red fish, so Betty must have had a blue fish. Think about it, then find a proof that mirrors how you think.</u>*

Resolve <u>( (¬ A) (¬ B) (¬ C) (¬ D) )</u> with <u>( (¬ C) D)</u> to produce: <u>( (¬ A) (¬ B) (¬ C ) )</u>

Resolve <u>( (¬ A) (¬ B) (¬ C ) )</u> with <u>( (¬ A) C)</u> to produce: <u>( (¬ A) (¬ B) )</u>

Resolve <u>( (¬ A) (¬ B) )</u> with <u>(B)</u> to produce: <u>(¬ A)</u>

Resolve <u>(A C)</u> with <u>(A (¬ C) )</u> to produce: <u>(A)</u>

Resolve <u>(¬ A)</u> with <u>(A)</u> to produce: <u>( )</u>

Other proofs are OK, provided they are correct.

Resolve ______

Resolve ______

Resolve ______

Resolve ______

Resolve ______

Resolve ______

**STRATEGY HINT: Always try to reduce the number of literals. Look for cases where the number of literals will decrease (eventually, you need to decrease the number of literals to zero!). Note that in every line in the proof above, the resolvent has fewer literals than in the longest clause that produced it. Look for cases where the two input clauses share other literals, which will be simplified. For example, on line #1 the literal (¬ C) is shared in both input clauses, so the net result is simply to cancel the (¬ D) in the first clause. Look for cases where one clause is a singleton, which <u>always</u> reduces the number of literals that result in the resolvent. For example, in line #3 the singleton clause (B) simply cancels the (¬ B) in the first clause. Look for opportunities to produce new singleton clauses, which can be used later to reduce the number of literals in other productions. For example, the singleton (¬ A) produced in line #3 is used later in line #5 to reduce the number of literals to zero, thereby achieving the goal ( ).**

Other proofs are OK provided that they are correct. For example, another correct proof is:

Resolve (A C) with (A (¬ C) ) to produce: (A)

Resolve (A C) with ( (¬ A) C) to produce: (C)

Resolve (C D) with ( (¬ C) D) to produce: (D)

Resolve ( (¬ A) (¬ B) (¬ C) (¬ D) ) with (A) to produce: ( (¬ B) (¬ C) (¬ D) )

Resolve ( (¬ B) (¬ C) (¬ D) ) with (B) to produce: ( (¬ C) (¬ D) )

Resolve ( (¬ C) (¬ D) ) with (C) to produce: (¬ D)

Resolve (¬ D) with (D) to produce: ( )

Resolve ______ with ______ to produce: ______

**These three true sentences state that Amy, Cindy, and Diane had the red fish.**

**The contradiction arises because B cannot be true in any world in which A, C, and D are true.**

**Note that in every line in the proof above, the resolvent has fewer literals than in the longest clause that produced it.**