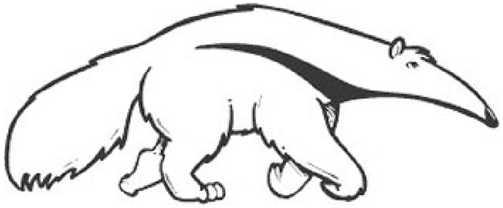
Uninformed Search

CS171, Winter 2018

Introduction to Artificial Intelligence

Prof. Richard Lathrop



Reading: R&N 3.1-3.4

Uninformed search strategies

- Uninformed (blind):
  - You have no clue whether one non-goal state is better than any other. Your search is blind. You don’t know if your current exploration is likely to be fruitful.

- Various blind strategies:
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Iterative deepening search (generally preferred)
  - Bidirectional search (preferred if applicable)

Search strategy evaluation

- A search strategy is defined by the order of node expansion

- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree (always finite)
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)
  - (for UCS: C^*: true cost to optimal goal; $\varepsilon > 0$: minimum step cost)

Uninformed search design choices

- Queue for Frontier:
  - FIFO? LIFO? Priority?

- Goal-Test:
  - Do goal-test when node inserted into *Frontier*?
  - Do goal-test when node removed?

- Tree Search, or Graph Search:
  - Forget *Expanded* (or *Explored*, Fig. 3.7) nodes?
  - Remember them?

Queue for Frontier

- FIFO (First In, First Out)
  - Results in Breadth-First Search
- LIFO (Last In, First Out)
  - Results in Depth-First Search
- Priority Queue sorted by path cost so far
  - Results in Uniform Cost Search
- Iterative Deepening Search uses Depth-First
- Bidirectional Search can use either Breadth-First or Uniform Cost Search

When to do goal test?

- Do Goal-Test when node is popped from queue

  IF you care about finding the optimal path

  AND your search space may have both short expensive and long cheap paths to a goal.
  - Guard against a short expensive goal.
  - E.g., Uniform Cost search with variable step costs.
- Otherwise, do Goal-Test when is node inserted.
  - E.g., Breadth-first Search, Depth-first Search, or Uniform Cost search when cost is a non-decreasing function of depth only (which is equivalent to Breadth-first Search).
- REASON ABOUT your search space & problem.
  - How could I possibly find a non-optimal goal?

General tree search

Goal test after pop

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;   ACTION[s] ← action;   STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

General graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Goal test after pop

Breadth-first graph search

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0 if
  problem.GOAL-TEST(node.STATE) then return SOLUTION(node) frontier ←
  a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)   /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
```

Goal test before push

Figure 3.11 Breadth-first search on a graph.

Uniform cost search

UCS: sort by g
GBFS: identical, use h
A*: identical, but use f = g+h

Goal test after pop

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier ← a priority queue ordered by PATH-COST, with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the lowest-cost node in frontier */
     if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           frontier ← INSERT(child, frontier)
        else if child.STATE is in frontier with higher PATH-COST then
           replace that frontier node with child
```

Figure 3.14 Uniform-cost search on a graph. The algorithm is identical to the general graph search algorithm in Figure 3.7, except for the use of a priority queue and the addition of an extra check in case a shorter path to a frontier state is discovered. The data structure for frontier needs to support efficient membership testing, so it should combine the capabilities of a priority queue and a hash table.

Depth-limited search & IDS

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Goal test before push

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

Checking for identical nodes

- It is “easy” to check the fringe/frontier
  - Keep a hash table holding all frontier nodes
    - Hash size is same O(n) as priority queue, so hash does not increase overall O(n)
  - When a node is expanded, remove it from hash
  - For each resulting child:
    - If child is not in hash, add it to queue and hash
    - Else if a lower-score node is in hash, discard the higher-score child
    - Else remove the higher-score node from queue and hash, and add the lower-score child to queue and hash
- It is memory-intensive to check explored/expanded
  - Keep a hash table holding all expanded nodes (may be HUGE!!)
  - When a node is expanded, add it to hash
    - For optimal searches, if already in hash, retain in hash the lower-score node
  - Discard any resulting child already in hash
    - For optimal searches, discard only if node in hash has lower score

Checking for search being in a loop

- It is "easy" to check for search being in a loop
  - When a node is expanded, for each child:
    - Trace back through parent pointers from child to root
    - If an ancestor is identical to the child, search is looping
      - Discard child and fail on that branch
    - Time complexity of child loop check is O(depth(child))
    - Memory consumption is zero
      - Assuming good garbage collection

When to do Goal-Test? Summary

- For DFS, BFS, DLS, and IDS, the goal test is done when the child node is generated.
  - These are not optimal searches in the general case.
  - BFS and IDS are optimal if cost is a function of depth only; then, optimal goals are also shallowest goals and so will be found first

- For GBFS the behavior is the same whether the goal test is done when the node is generated or when it is removed
  - h(goal)=0 so any goal will be at the front of the queue anyway.

- For UCS and A* the goal test is done when the node is removed from the queue.
  - This precaution avoids finding a short expensive path before a long cheap path.

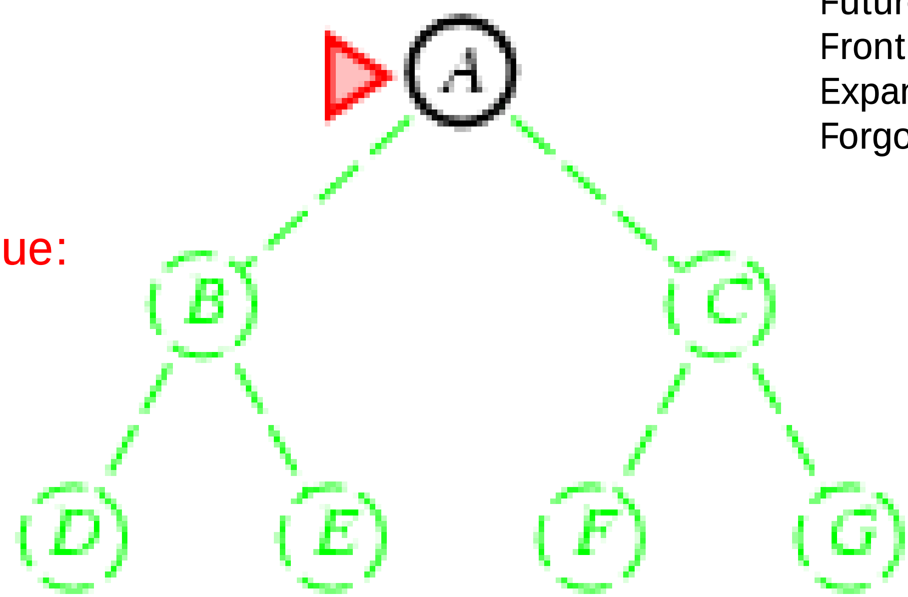
Breadth-first search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored
  - also called Fringe, or OPEN
- Implementation:
  - *Frontier* is a first-in-first-out (FIFO) queue (new successors go at end)
  - Goal test when inserted

Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

Initial state = A
Is A a goal state?

Put A at end of queue:
Frontier = [A]



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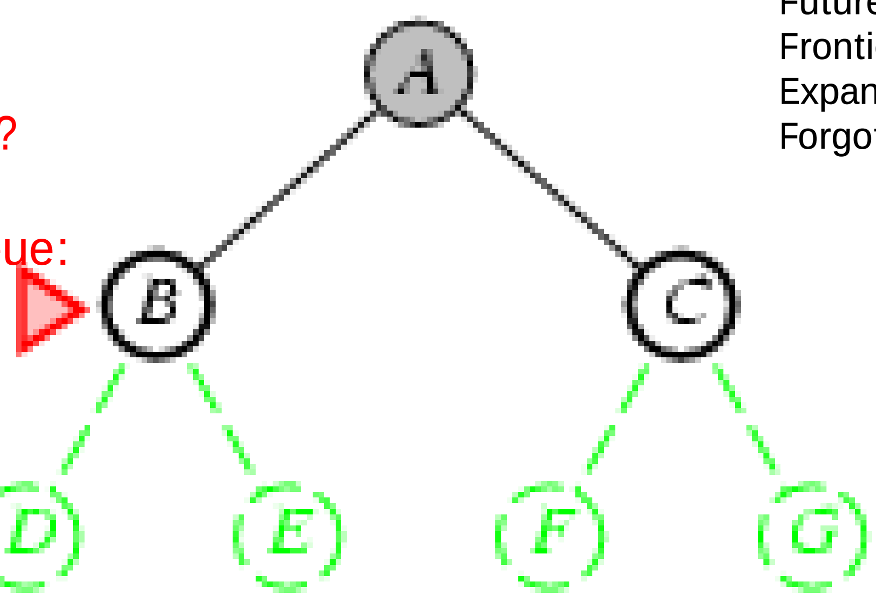
Expand A to B,C
Is B or C a goal state?

Put B,C at end of queue:
Frontier = [B,C]



Breadth-first search

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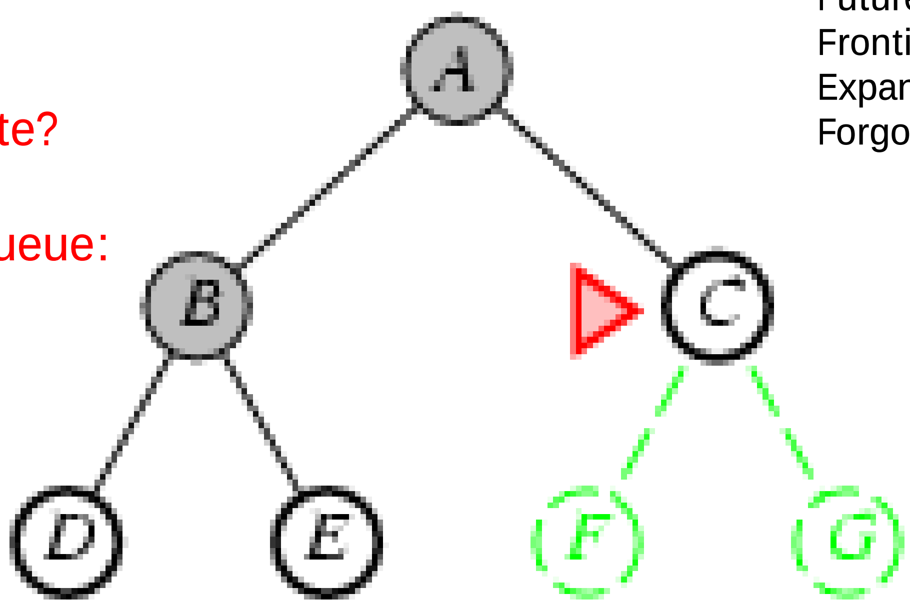
Expand B to D,E
Is D or E a goal state?

Put D,E at end of queue:
Frontier = [C,D,E]



Breadth-first search

- Expand shallowest unexpanded node
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- Implementation:
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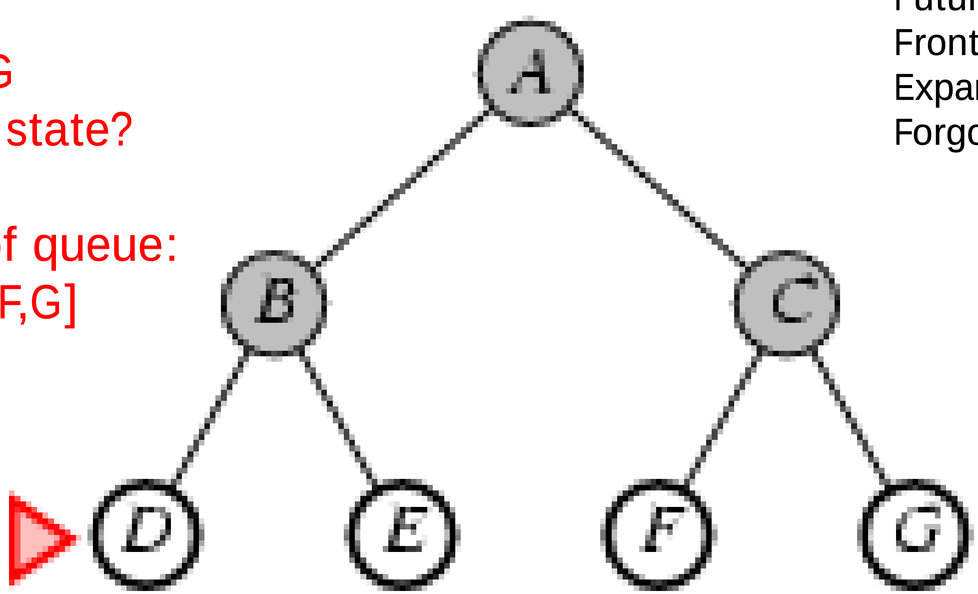
Expand C to F, G
Is F or G a goal state?

Put F,G at end of queue:
Frontier = [D,E,F,G]



Breadth-first search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored
  - also called Fringe, or OPEN
- Implementation:
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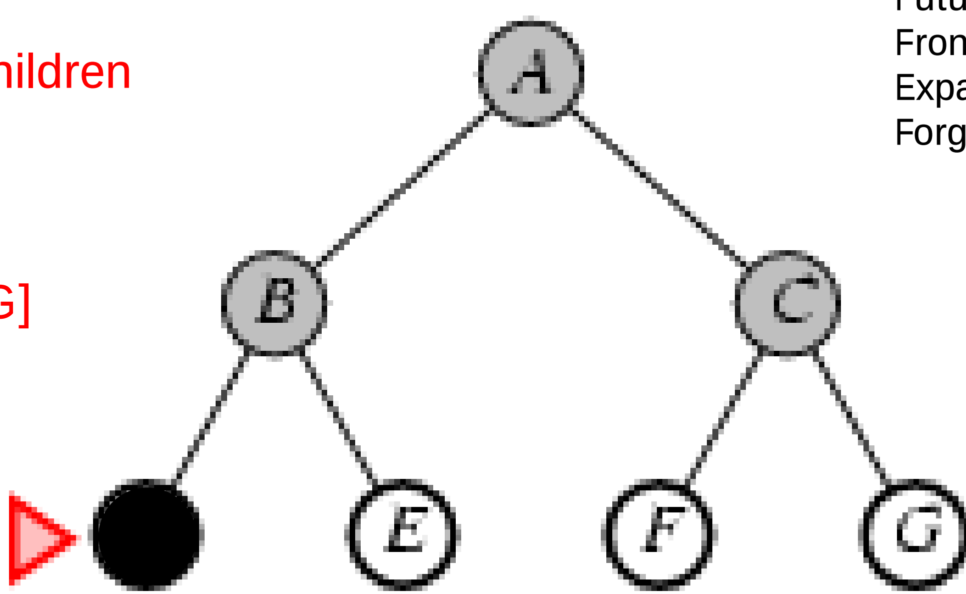
  - Goal test when inserted

Expand D; no children
Forget D

Frontier = [E,F,G]



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Breadth-first search

- Expand shallowest unexpanded node
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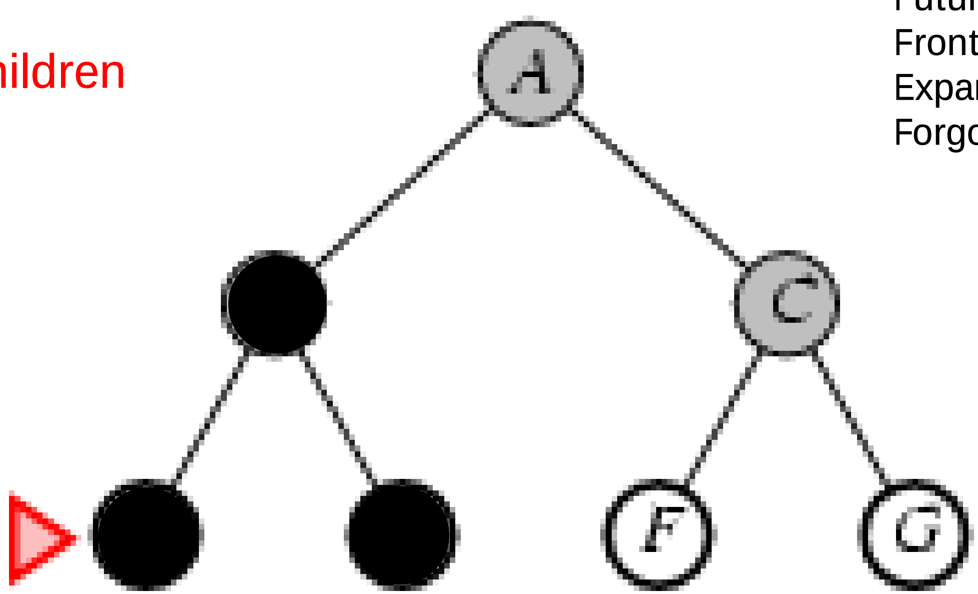
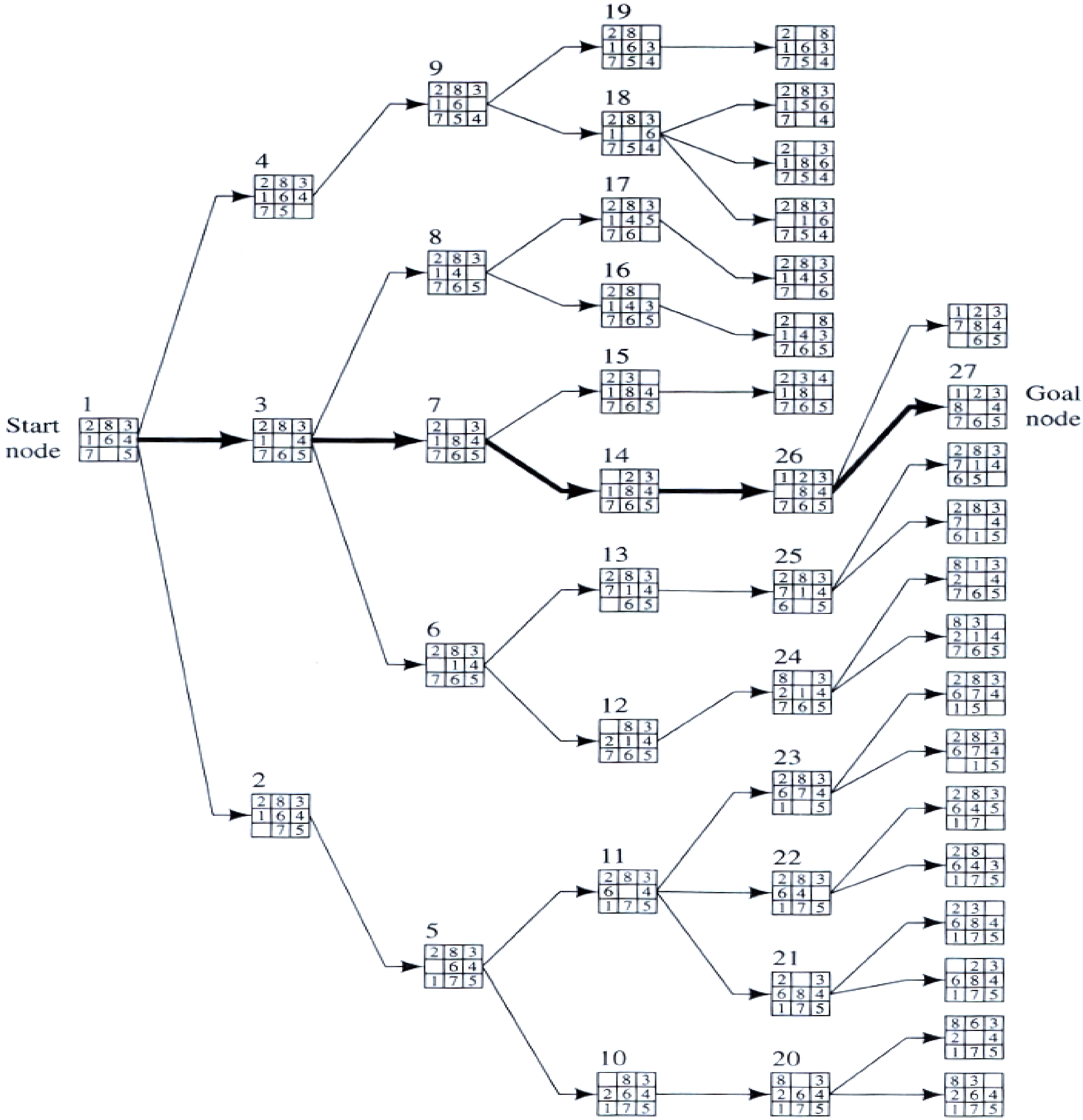
Expand E; no children
Forget E; B

Frontier = [F,G]



Example BFS for 8-puzzle



Properties of breadth-first search

- Complete? Yes, it always reaches a goal (if *b* is finite)
- Time? $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$

  (this is the number of nodes we generate)
- Space? $O(b^d)$

  (keeps every node in memory, either in frontier or on a path to frontier).
- Optimal? No, for general cost functions.

  Yes, if cost is a non-decreasing function only of depth.
  - With $f(d) \geq f(d-1)$, e.g., step-cost = constant:
    - All optimal goal nodes occur on the same level
    - Optimal goals are always shallower than non-optimal goals
    - An optimal goal will be found before any non-optimal goal

- Usually Space is the bigger problem (more than time)

BFS: Time & Memory Costs

Depth of Solution	Nodes Expanded	Time	Memory
0	1	5 microseconds	100 bytes
2	111	0.5 milliseconds	11 kbytes
4	11,111	0.05 seconds	1 megabyte
8	10^8	9.25 minutes	11 gigabytes
12	10^{12}	64 days	111 terabytes

Assuming b=10; 200k nodes/sec; 100 bytes/node

Uniform-cost search

Breadth-first is only optimal if path cost is a non-decreasing function of depth, i.e., $f(d) \geq f(d-1)$; e.g., constant step cost, as in the 8-puzzle.

Can we guarantee optimality for variable positive step costs $\geq \varepsilon$?

(Why $\geq \varepsilon$? To avoid infinite paths w/ step costs 1, ½, ¼, …)

Uniform-cost Search:

Expand node with smallest path cost g(n).

- *Frontier* is a priority queue, i.e., new successors are merged into the queue sorted by g(n).
  - Can remove successors already on queue w/higher g(n).
    - Saves memory, costs time; another space-time trade-off.
- *Goal-Test* when node is popped off queue.

Uniform-cost search

Implementation: *Frontier* = queue ordered by path cost.

Equivalent to breadth-first if all step costs all equal.

- Complete? Yes, if b is finite and step cost $\geq \varepsilon > 0$.

  (otherwise it can get stuck in infinite loops)

- Time? # of nodes with path cost $\leq$ cost of optimal solution.

  $O(b^{\lfloor 1+C^*/\varepsilon \rfloor}) \approx O(b^{d+1})$

- Space? # of nodes with path cost $\leq$ cost of optimal solution.

  $O(b^{\lfloor 1+C^*/\varepsilon \rfloor}) \approx O(b^{d+1})$.

- Optimal? Yes, for any step cost $\geq \varepsilon > 0$.

Uniform-cost search

Proof of Completeness:

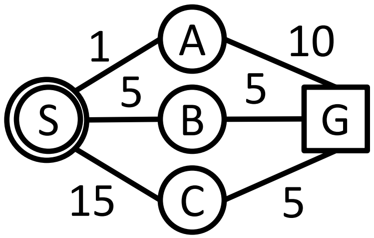
Assume (1) finite max branching factor = *b;* (2) min step cost $\geq \varepsilon > 0$; (3) cost to optimal goal = *C**. Then a node at depth $\lfloor 1+C^*/\varepsilon \rfloor$ must have a path cost > C*. There are O(b^($\lfloor 1+C^*/\varepsilon \rfloor$) such nodes, so a goal will be found.

Proof of Optimality (given completeness):

Suppose that UCS is not optimal. Then there must be an (optimal) goal state with path cost smaller than the found (suboptimal) goal state (invoking completeness). However, this is impossible because UCS would have expanded that node first, by definition. Contradiction.

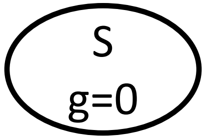
Ex: Uniform-cost search

(Search tree version)



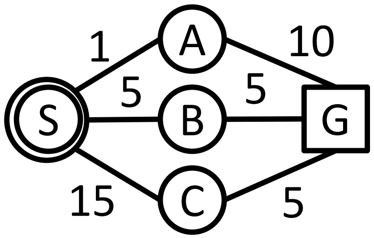
Route finding problem.
Steps labeled w/cost.

Order of node expansion: ______________________________
Path found: ________________ Cost of path found: _________



Ex: Uniform-cost search

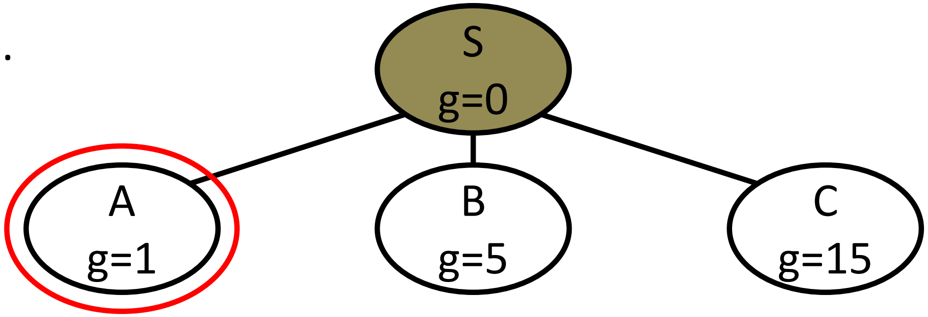
(Search tree version)



Route finding problem.
Steps labeled w/cost.

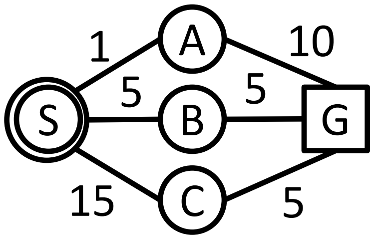
Order of node expansion: S

Path found: ________________ Cost of path found: ________



Ex: Uniform-cost search

(Search tree version)

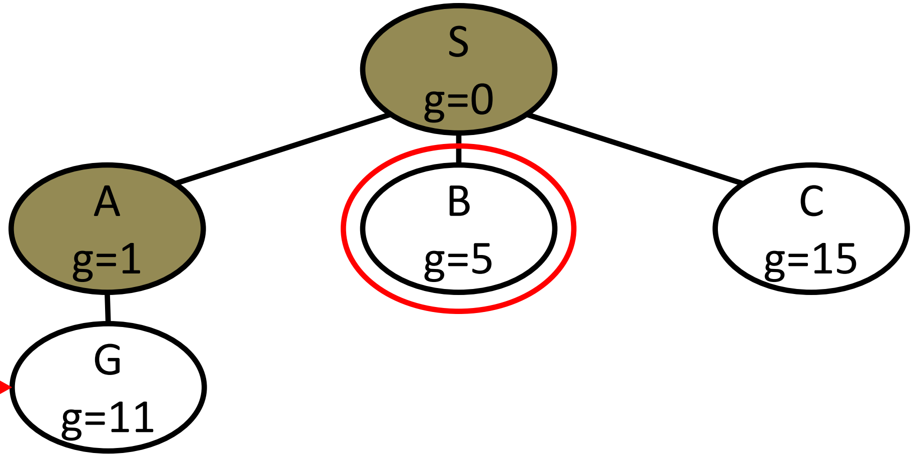


Order of node expansion: S A

Path found: ________________ Cost of path found: ________

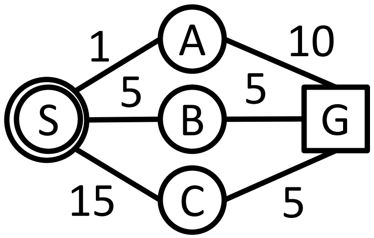
Route finding problem.
Steps labeled w/cost.

This early expensive goal node will go back onto the queue until after the later cheaper goal is found.



Ex: Uniform-cost search

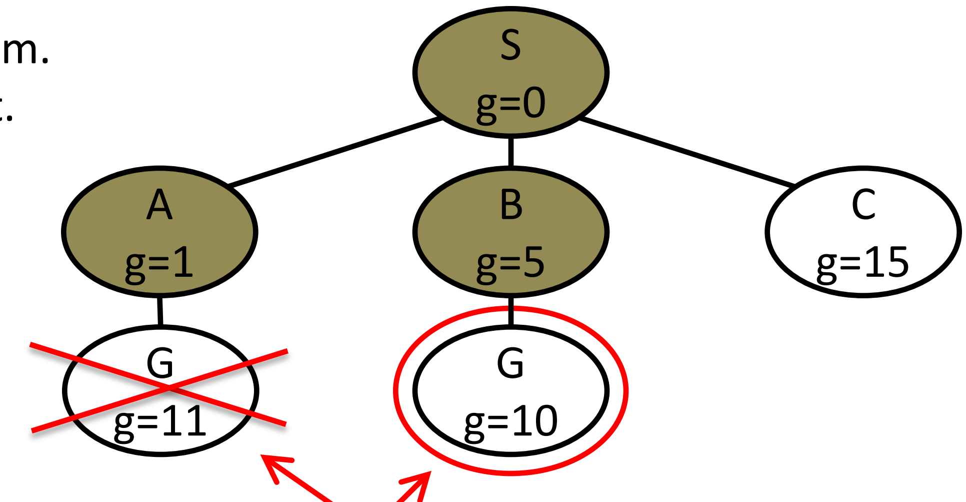
(Search tree version)



Route finding problem.
Steps labeled w/cost.

Order of node expansion: S A B ____________

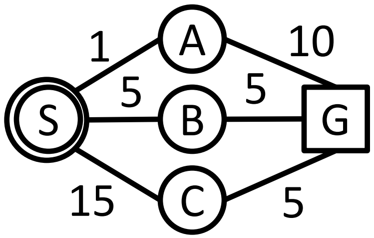
Path found: ____________ Cost of path found: ________



Remove the higher-cost of identical nodes and save memory. However, UCS is optimal even if this is not done, since lower-cost nodes sort to the front.

Ex: Uniform-cost search

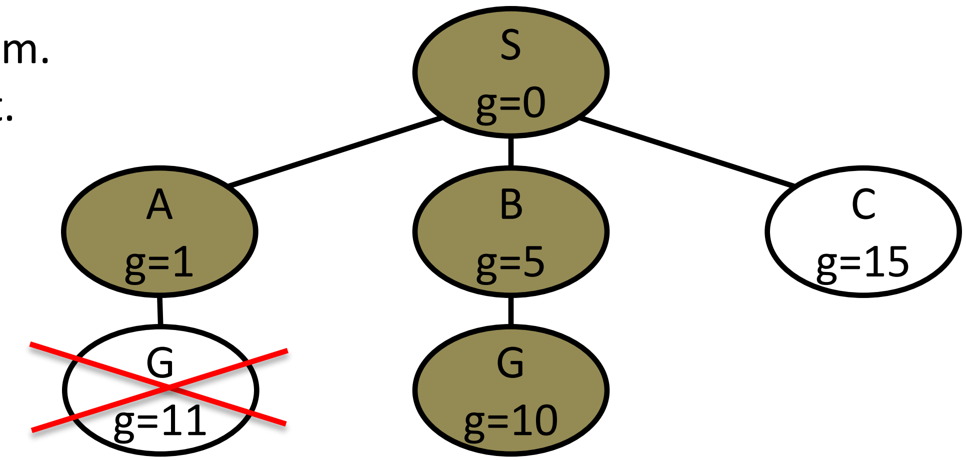
(Search tree version)



Route finding problem.
Steps labeled w/cost.

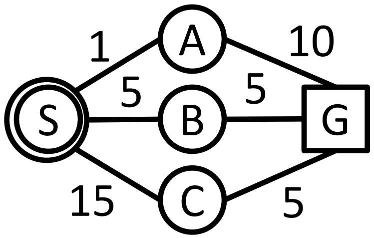
Order of node expansion: S A B G

Path found: S B G   Cost of path found: 10



Technically, the goal node is not really expanded, because we do not generate the children of a goal node. It is listed in "Order of node expansion" only for your convenience, to see explicitly where it was found.

Ex: Uniform-cost search (Virtual queue version)



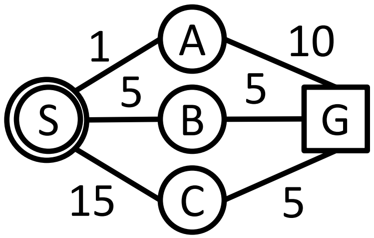
Order of node expansion: ___________________________

Path found: ______________ Cost of path found: _______

Route finding problem.
Steps labeled w/cost.

Expanded:
Next:
Children:
Queue: S/g=0

Ex: Uniform-cost search (Virtual queue version)



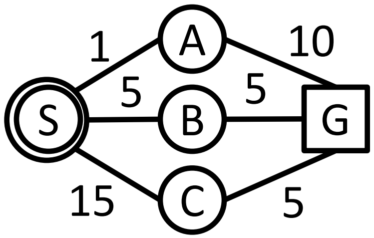
Order of node expansion: ___S____________________________

Path found: __________________ Cost of path found: _________

Route finding problem.
Steps labeled w/cost.

Expanded: S/g=0
Next: S/g=0
Children: A/g=1, B/g=5, C/g=15
Queue: S/g=0, A/g=1, B/g=5, C/g=15

Ex: Uniform-cost search (Virtual queue version)



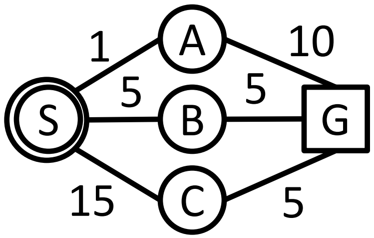
Route finding problem.
Steps labeled w/cost.

Order of node expansion: S A
Path found: ________________ Cost of path found: ________

Expanded: S/g=0, A/g=1
Next: A/g=1
Children: G/g=11
Queue: S/g=0, A/g=1, B/g=5, C/g=15, G/g=11

Note that in a proper priority queue in a computer system, this queue would be sorted by g(n). For hand-simulated search it is more convenient to write children as they occur, and then scan the current queue to pick the highest-priority node on the queue.

Ex: Uniform-cost search (Virtual queue version)



Route finding problem.
Steps labeled w/cost.

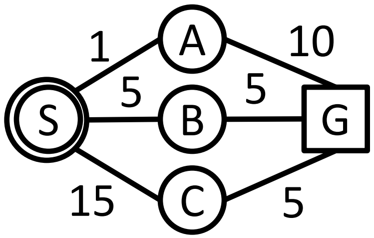
Order of node expansion: ___S A B___________________
Path found: _________________ Cost of path found: _________

Expanded: S/g=0, A/g=1, B/g=5
Next: B/g=5
Children: G/g=10
Queue: S/g=0, A/g=1, B/g=5, C/g=15, ~~G/g=11~~, G/g=10

Remove the higher-cost of identical nodes and save memory. However, UCS is optimal even if this is not done, since lower-cost nodes sort to the front.

Ex: Uniform-cost search

(Virtual queue version)



Route finding problem.
Steps labeled w/cost.

Order of node expansion: S A B G

Path found: S B G   Cost of path found: 10

The same “Order of node expansion”, “Path found”, and “Cost of path found” is obtained by both methods. They are formally equivalent to each other in all ways.

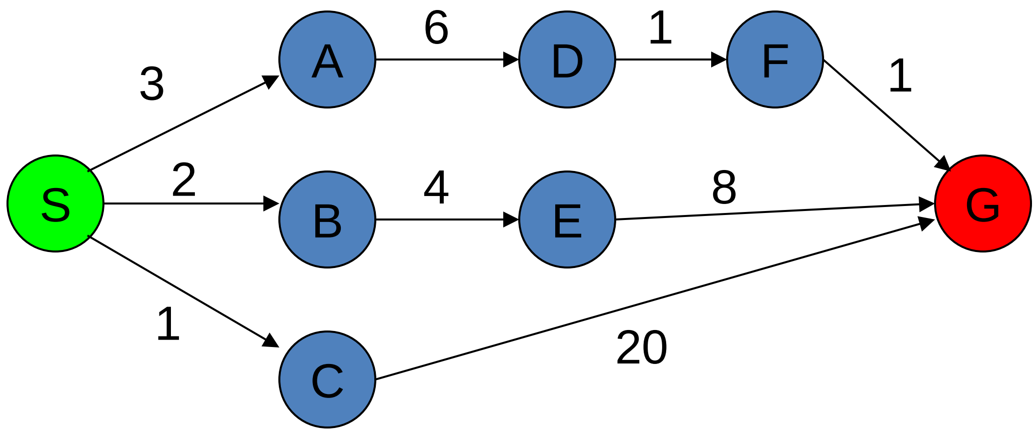
Expanded: S/g=0, A/g=1, B/g=5, G/g=10

Next: G/g=10

Children: none

Queue: S/g=0, A/g=1, B/g=5, C/g=15, ~~C/g=11~~, G/g=10

Technically, the goal node is not really expanded, because we do not generate the children of a goal node. It is listed in “Order of node expansion” only for your convenience, to see explicitly where it was found.



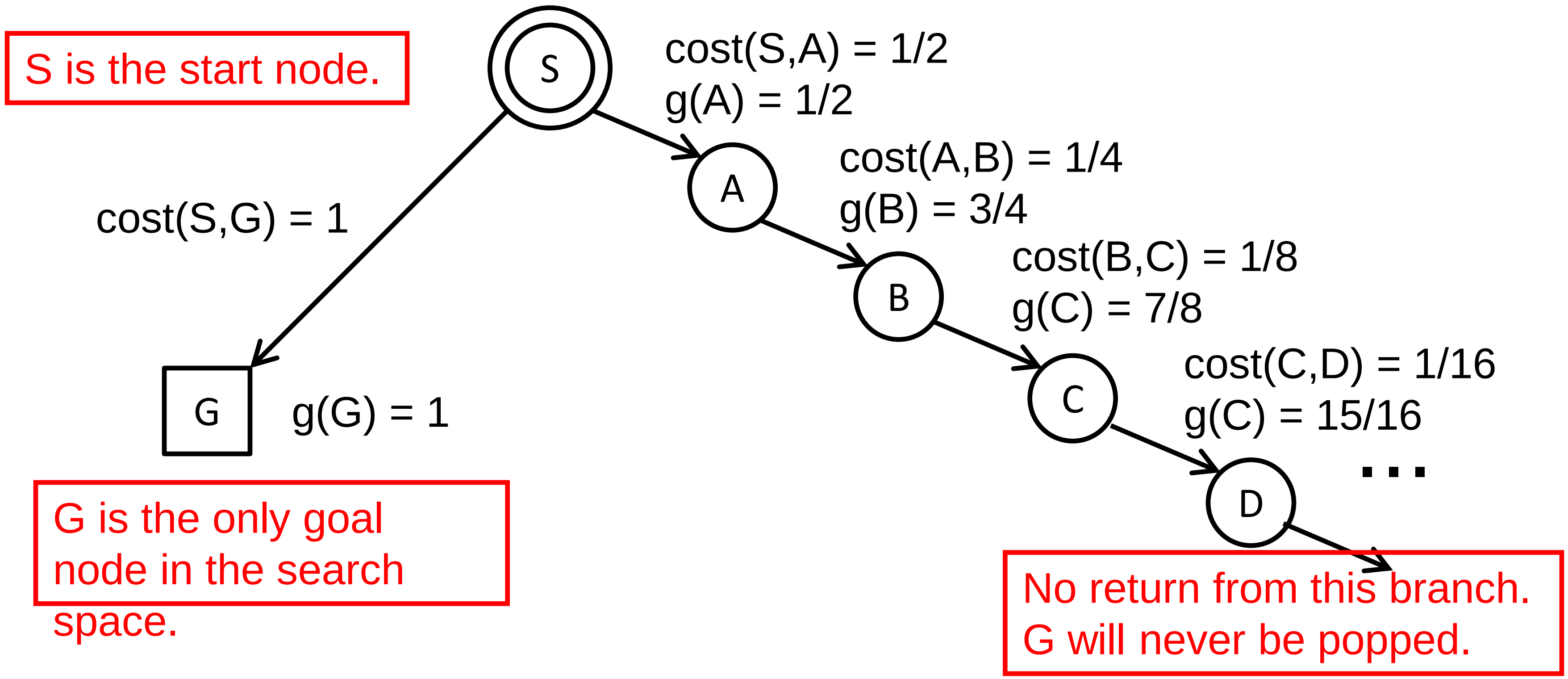
The graph above shows the step-costs for different paths going from the start (S) to the goal (G).

Use uniform cost search to find the optimal path to the goal.

Exercise for home

Uniform cost search

- Why require step cost $\geq \varepsilon > 0$?
  - Otherwise, an infinite regress is possible.
  - Recall: $\sum_{n=1}^{\infty} 2^{-n} = 1$



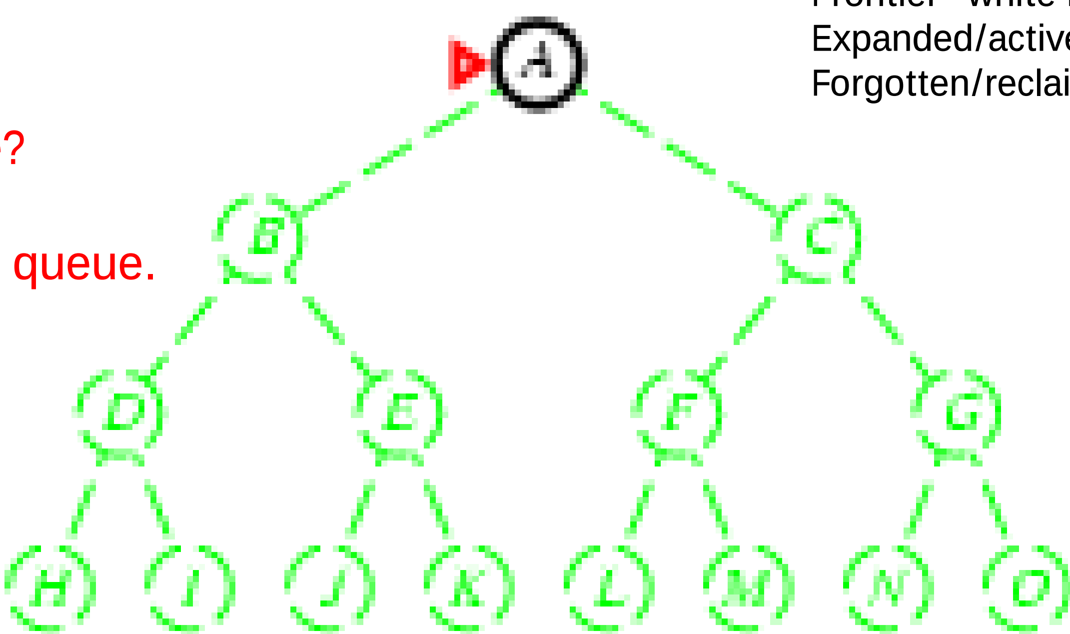
Depth-first search

- Expand *deepest* unexpanded node
- *Frontier* = Last In First Out (LIFO) queue, i.e., new successors go at the front of the queue.
- *Goal-Test* when inserted.

Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

Initial state = A
Is A a goal state?

Put A at front of queue.
frontier = [A]

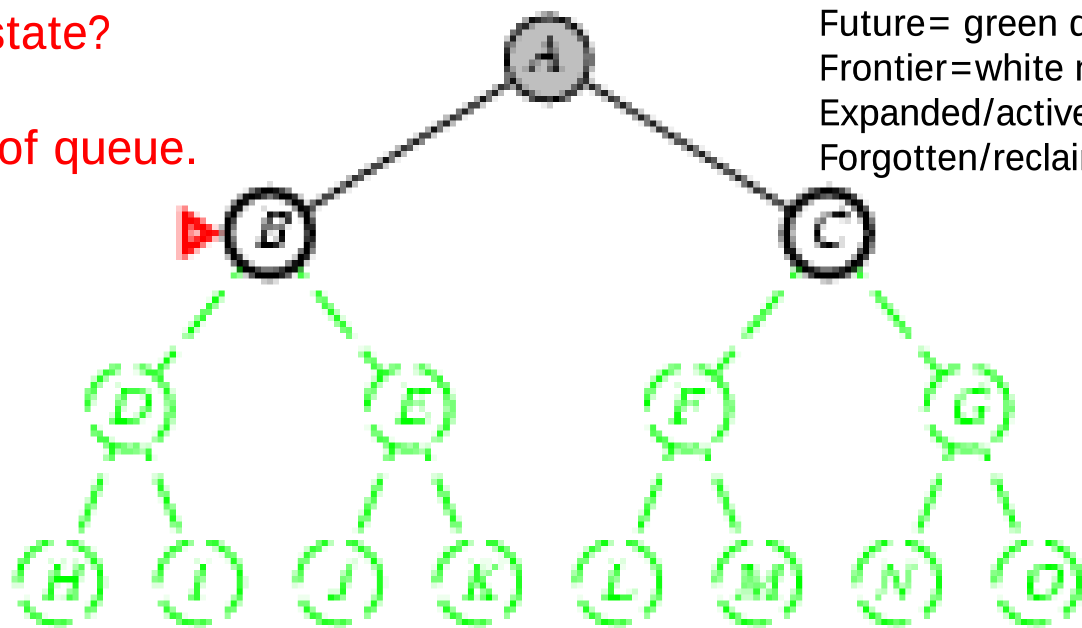


Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand A to B, C.
Is B or C a goal state?

Put B, C at front of queue.
frontier = [B,C]



Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

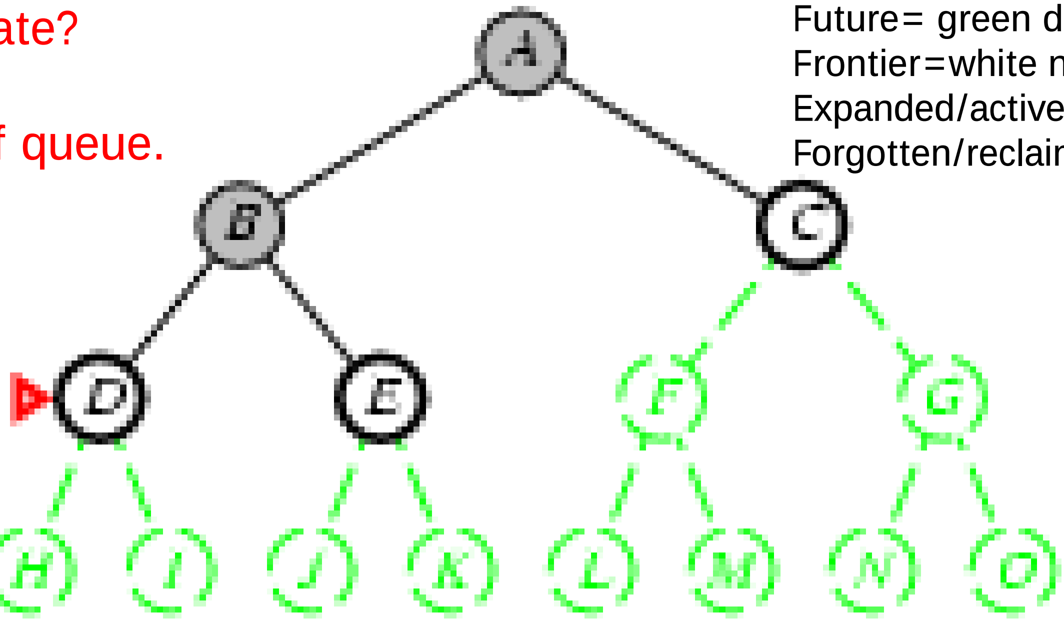
Note: Can save a space factor of *b* by generating successors one at a time.
See **backtracking search** in your book, p. 87 and Chapter 6.

Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand B to D, E.
Is D or E a goal state?

Put D, E at front of queue.
frontier = [D,E,C]



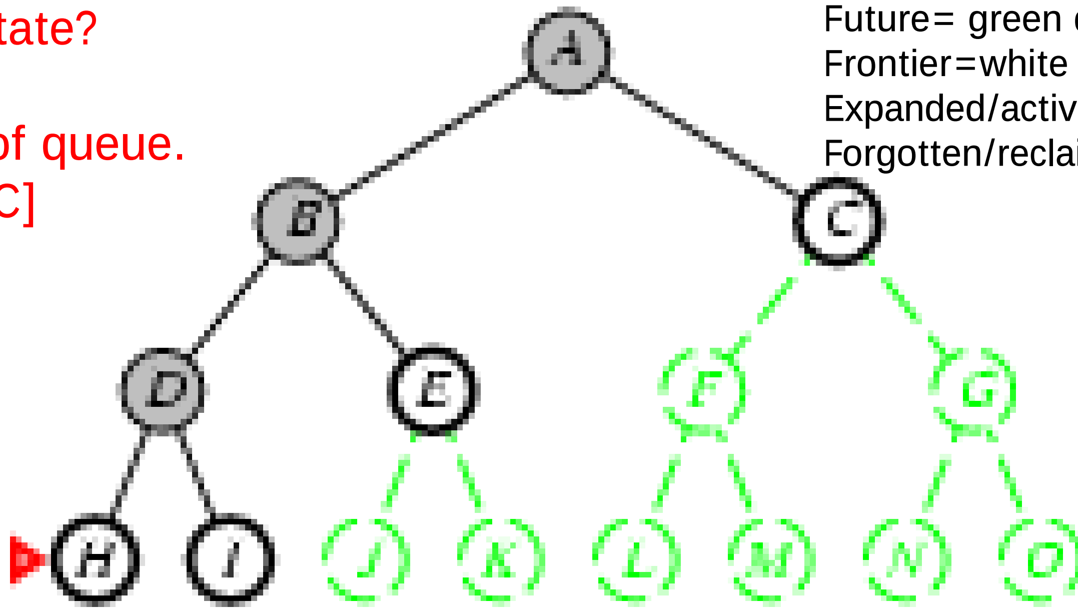
Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand D to H, I.
Is H or I a goal state?

Put H, I at front of queue.
frontier = [H,I,E,C]



Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

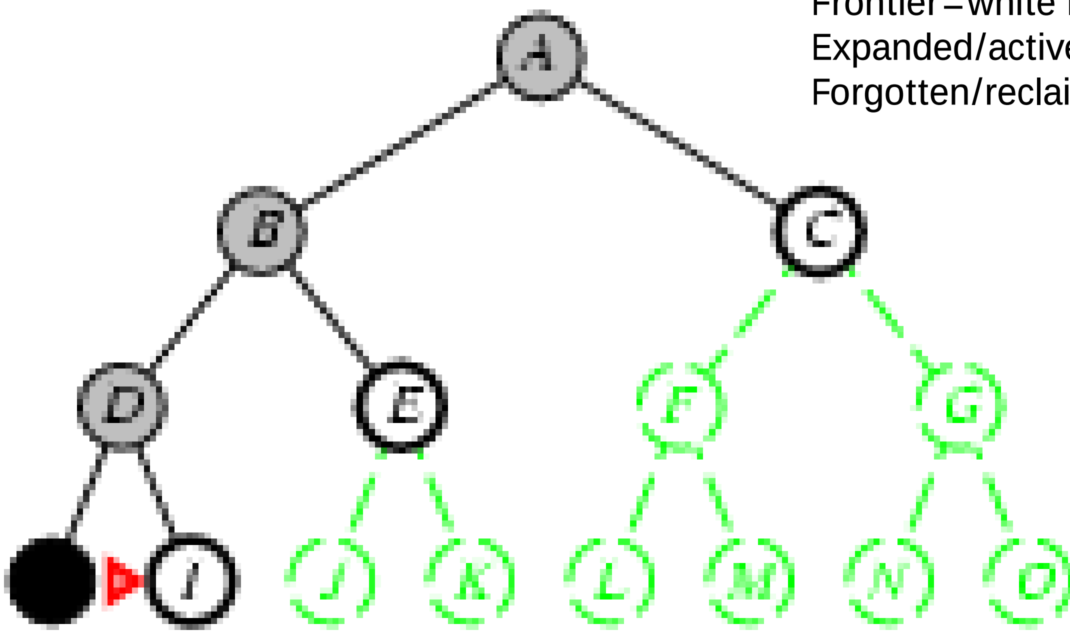
Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand H to no children.
Forget H.

frontier = [I,E,C]

Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

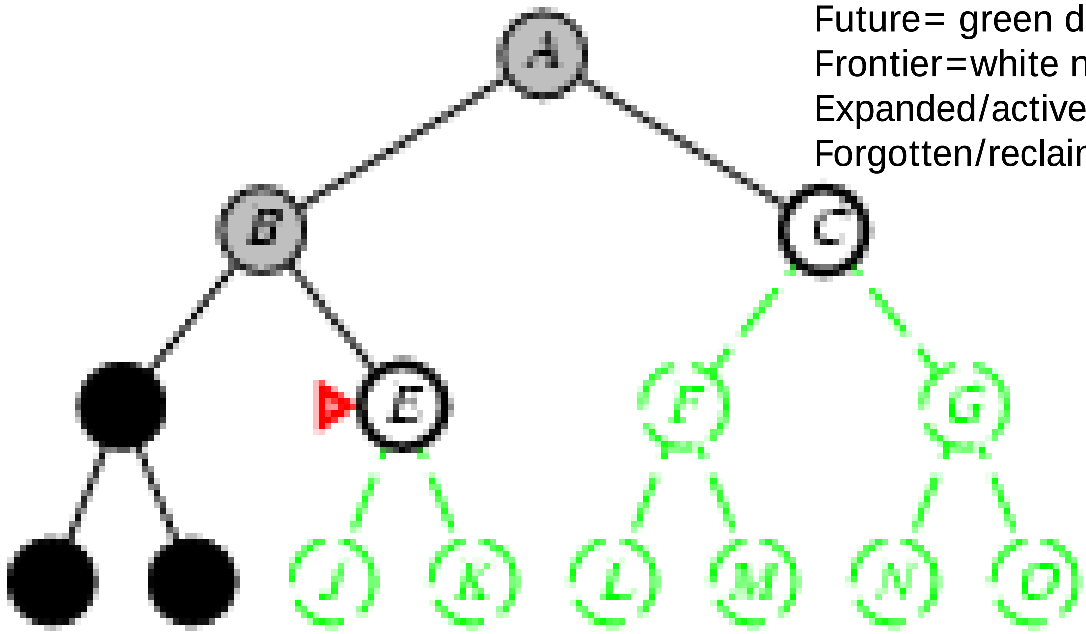


Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand I to no children.
Forget D, I.

frontier = [E,C]



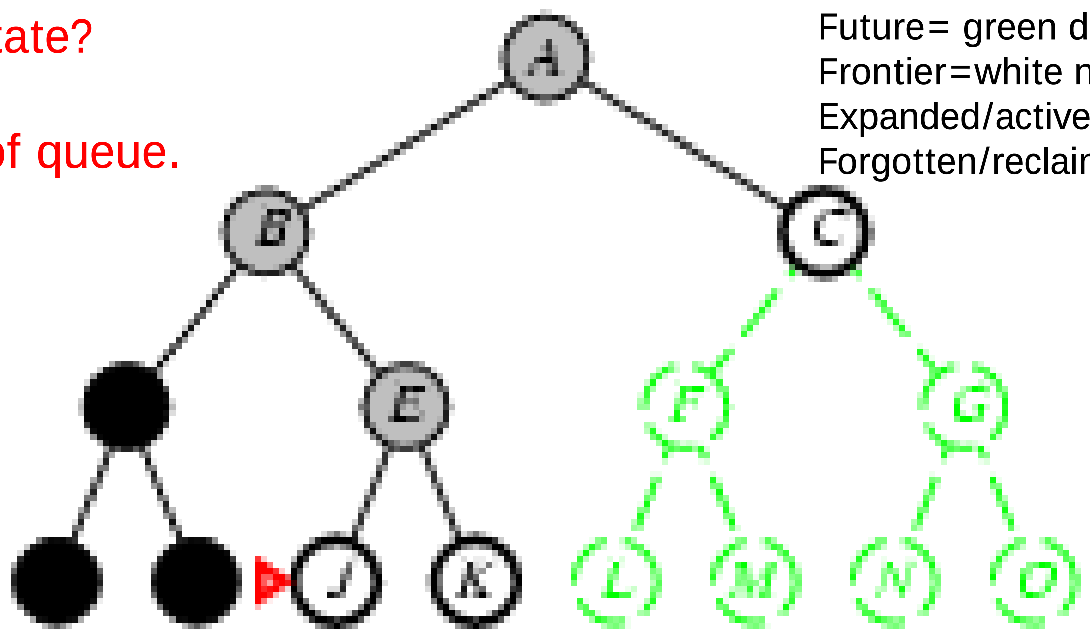
Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand E to J, K.
Is J or K a goal state?

Put J, K at front of queue.
frontier = [J,K,C]



Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

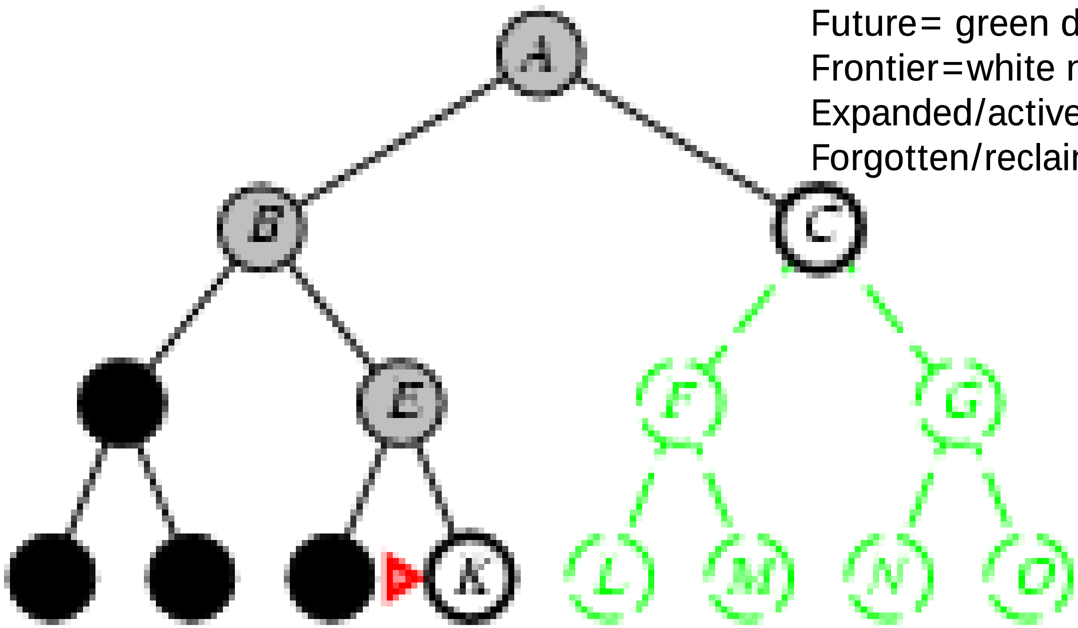
Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand J to no children.
Forget J.

frontier = [K,C]

Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

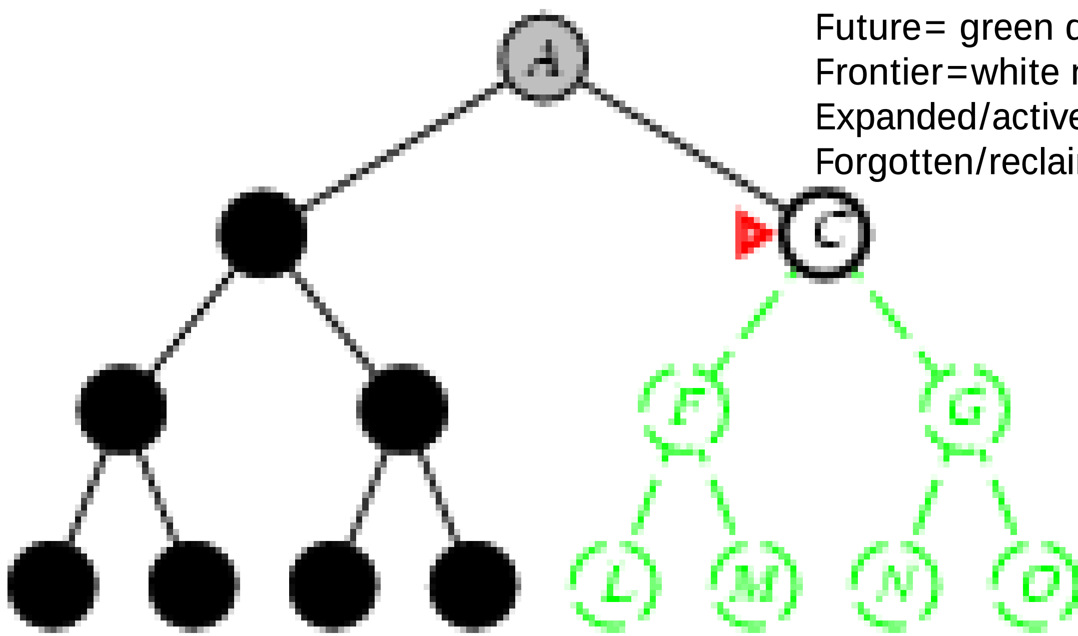


Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

Expand K to no children.
Forget B, E, K.

frontier = [C]



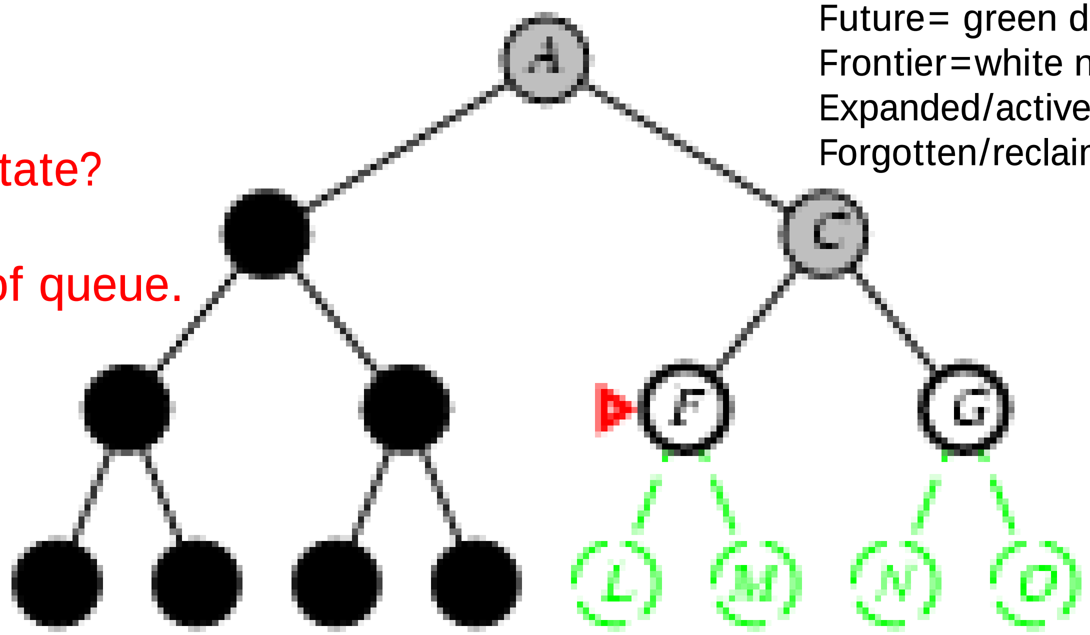
Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

Depth-first search

- Expand deepest unexpanded node
  - *Frontier* = LIFO queue, i.e., put successors at front

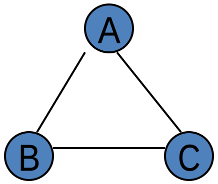
Expand C to F, G.
Is F or G a goal state?

Put F, G at front of queue.
frontier = [F,G]



Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes

Properties of depth-first search



- Complete? No: fails in loops/infinite-depth spaces
  - Can modify to avoid loops/repeated states along path
    - check if current nodes occurred before on path to root
  - Can use graph search (remember all nodes ever seen)
    - problem with graph search: space is exponential, not linear
  - Still fails in infinite-depth spaces (may miss goal entirely)

- Time? $O(b^m)$ with m =maximum depth of space
  - Terrible if m is much larger than d
  - If solutions are dense, may be much faster than BFS

- Space? $O(bm)$, i.e., linear space!
  - Remember a single path + expanded unexplored nodes

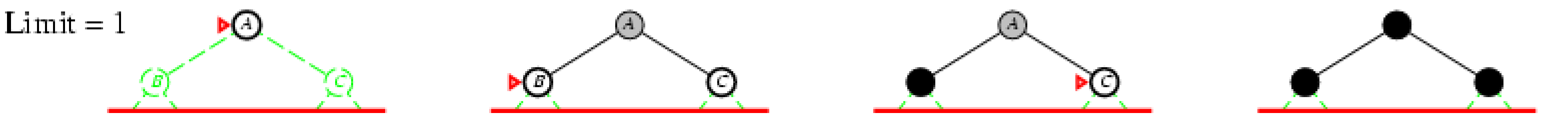
- Optimal? No: It may find a non-optimal goal first

Iterative Deepening Search

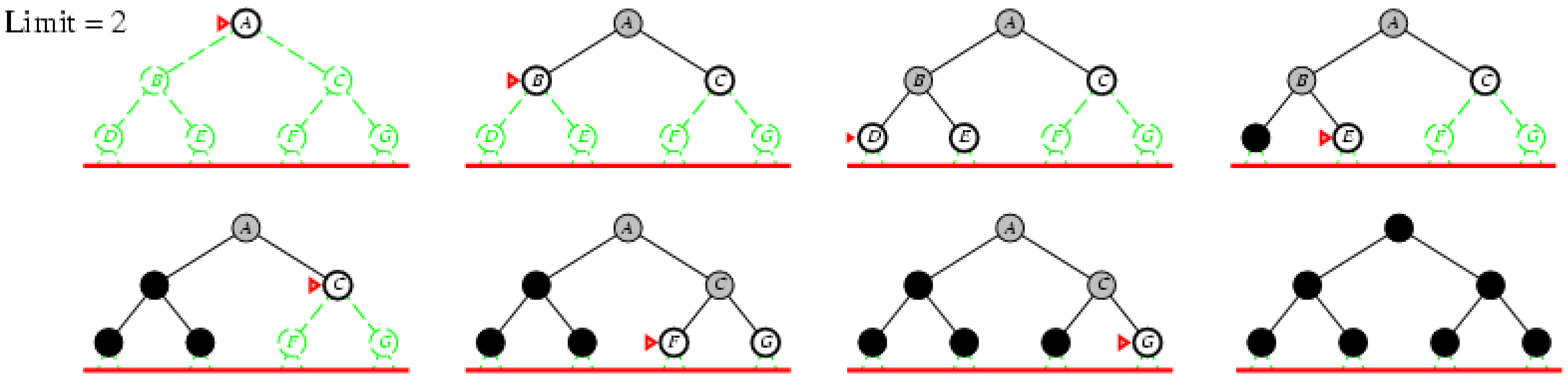
- To avoid the infinite depth problem of DFS:
  - Only search until depth L
  - i.e, don't expand nodes beyond depth L
  - Depth-Limited Search
- What if solution is deeper than L?
  - Increase depth iteratively
  - Iterative Deepening Search
- IDS
  - Inherits the memory advantage of depth-first search
  - Has the completeness property of breadth-first search

Iterative Deepening Search,   L=0

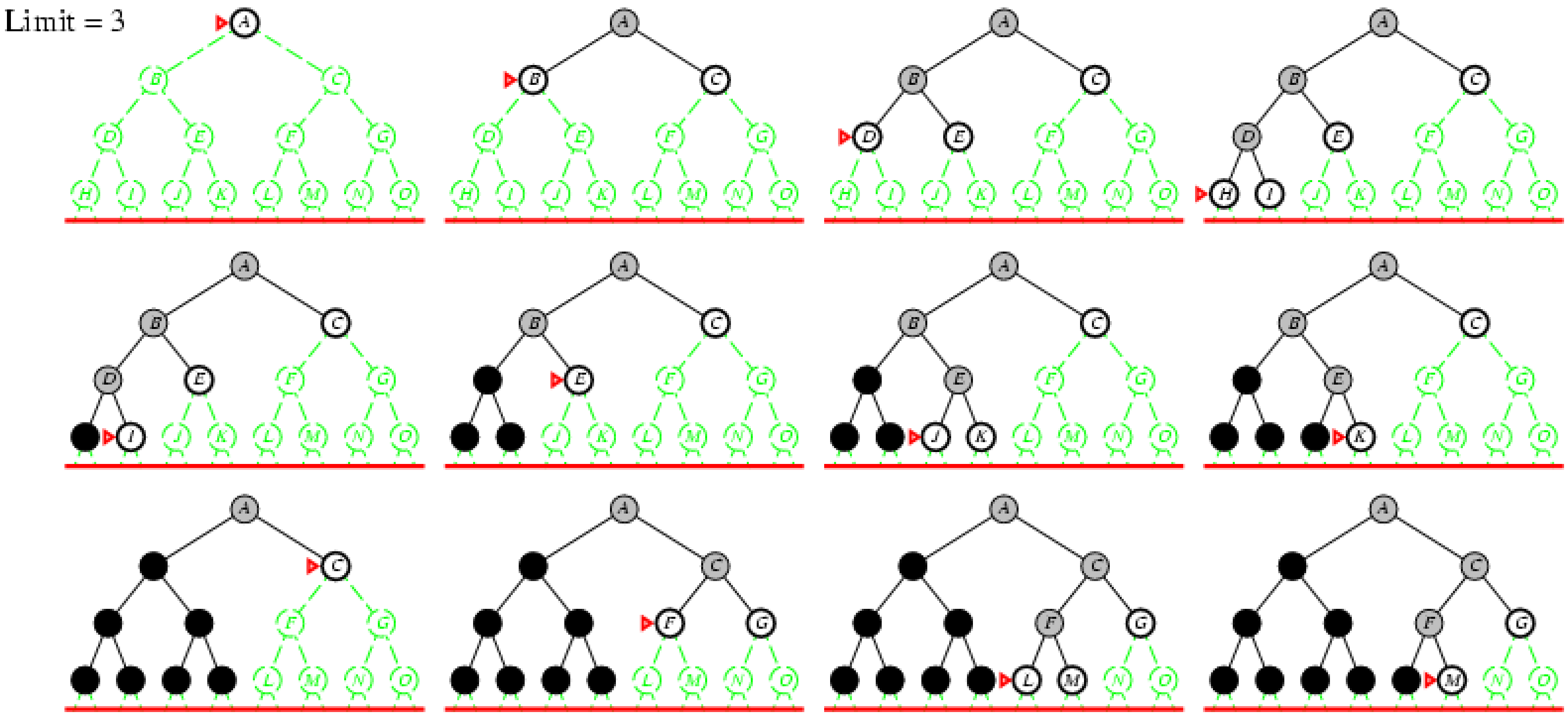
Iterative Deepening Search,   L=1



Iterative Deepening Search, L=2



Iterative Deepening Search,   L=3



Iterative Deepening Search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \dots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\, b^1 + (d-1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d$$
$$= O(b^d)$$

- For $b = 10$, $d = 5$,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}450$

[Ratio: b/(b-1)]

Properties of iterative deepening search

- Complete? Yes
- Time? $O(b^d)$
- Space? *O(bd)*
- Optimal? No, for general cost functions.
  Yes, if cost is a non-decreasing function only of depth.

Generally the preferred uninformed search strategy.

Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both "meet in the middle"
  - need to keep track of the intersection of 2 open sets of nodes

- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - what if there are multiple goal states?
  - what if there is only a goal test, no explicit list?

- Complexity
  - time complexity is best: $O(2\ b^{(d/2)}) = O(b^{(d/2)})$
  - memory complexity is the same as time complexity

Bi-Directional Search

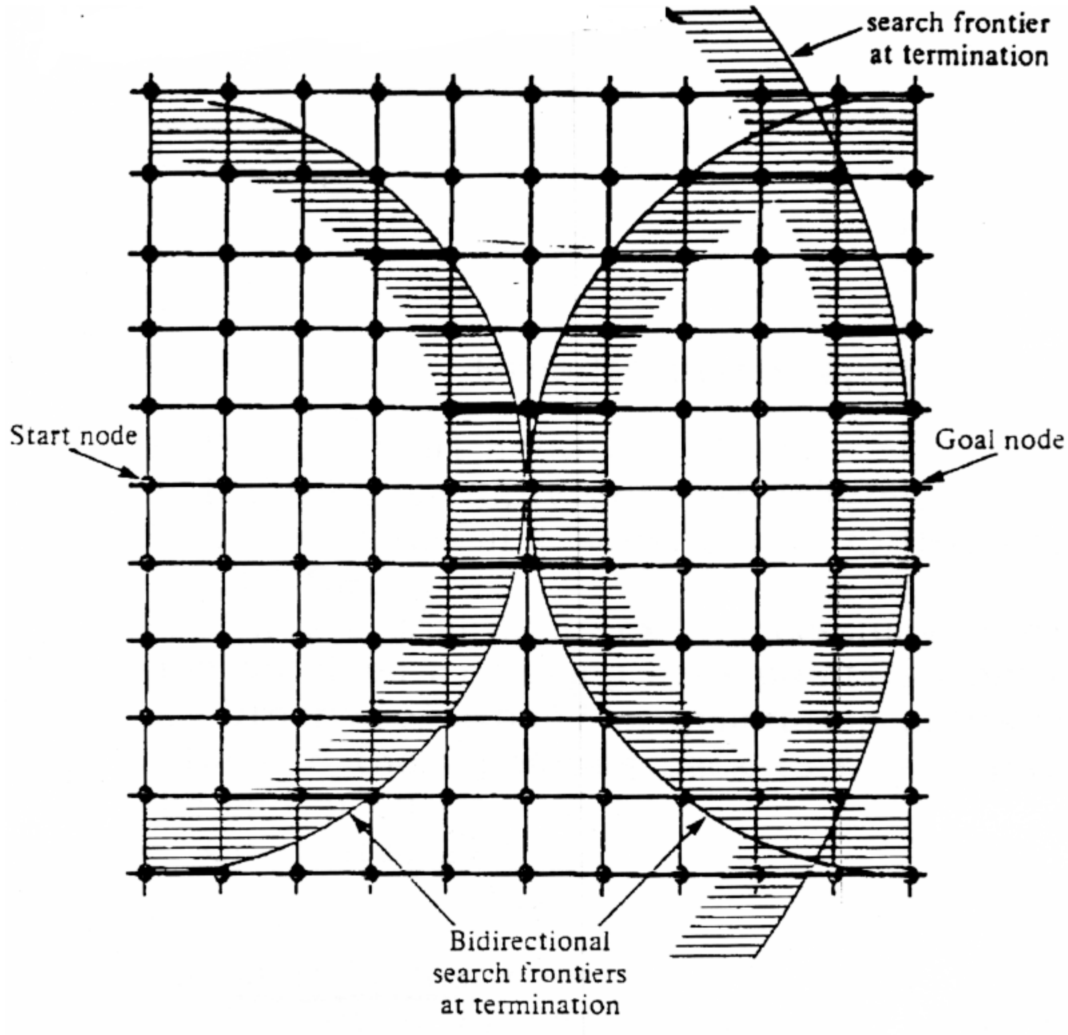


Fig. 2.10 Bidirectional and unidirectional breadth-first searches.

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening DLS	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{\lfloor 1+C^*/\varepsilon \rfloor})$	$O(b^m)$	$O(b^l)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{\lfloor 1+C^*/\varepsilon \rfloor})$	$O(bm)$	$O(bl)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

There are a number of footnotes, caveats, and assumptions.
See Fig. 3.21, p. 91.

[a] complete if b is finite
[b] complete if step costs $\geq \varepsilon > 0$
[c] optimal if step costs are all identical
(also if path cost non-decreasing function of depth only)
[d] if both directions use breadth-first search
(also if both directions use uniform-cost search with step costs $\geq \varepsilon > 0$)

Generally the preferred uninformed search strategy

Note that $d \leq \lfloor 1+C^*/\varepsilon \rfloor$

You should know…

- Overview of uninformed search methods
- Search strategy evaluation
  - Complete? Time? Space? Optimal?
  - Max branching (b), Solution depth (d), Max depth (m)
  - (for UCS: C*: true cost to optimal goal; $\varepsilon > 0$: minimum step cost)
- Search Strategy Components and Considerations
  - Queue? Goal Test when? Tree search vs. Graph search?
- Various blind strategies:
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Iterative deepening search (generally preferred)
  - Bidirectional search (preferred if applicable)

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

http://www.cs.rmit.edu.au/AI-Search/Product/
http://aima.cs.berkeley.edu/demos.html (for more demos)