CS-271P Final Review

- **Propositional Logic**
  - (7.1-7.5)
- **First-Order Logic, Knowledge Representation**
  - (8.1-8.5, 9.1-9.2)
- **Constraint Satisfaction Problems**
  - (6.1-6.4, except 6.3)
- **Machine Learning**
  - (18.1-18.4)
- Questions on any topic
- Pre-mid-term material if time and class interest
- Please review your quizzes, mid-term, & old tests
  - At least one question from a prior quiz or test will appear on the Final Exam (and all other tests)

Review Propositional Logic Chapter 7.1-7.5

- Definitions:
  - Syntax, Semantics, Sentences, Propositions, Entails, Follows, Derives, Inference, Sound, Complete, Model, Satisfiable, Valid (or Tautology)
- Syntactic Transformations:
  - E.g., $(A \Rightarrow B) \Leftrightarrow (\neg A \vee B)$
- Semantic Transformations:
  - E.g., $(KB \models \alpha) \equiv (\models (KB \Rightarrow \alpha)$
- Truth Tables:
  - Negation, Conjunction, Disjunction, Implication, Equivalence (Biconditional)
- Inference:
  - By Model Enumeration (truth tables)
  - By Resolution

Recap propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas

- The proposition symbols P_1, P_2 etc are sentences

  - If S is a sentence, $\neg$S is a sentence (negation)
  - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Recap propositional logic:

Semantics

Each model/world specifies true or false for each proposition symbol

E.g., $P_{1,2}$ $P_{2,2}$ $P_{3,1}$

false true false

With these symbols, 8 possible models can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	$S_1$is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
(i.e.,	is false iff	S_1 is true and	S_2 is false)
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true	

Simple recursive process evaluates an arbitrary sentence, e.g.,

$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$

Recap propositional logic:
Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

OR: P or Q is true or both are true.
XOR: P or Q is true but not both.

Implication is always true when the premises are False!

Recap propositional logic:

Logical equivalence and rewrite rules

- To manipulate logical sentences we need some rewrite rules.
- Two sentences are logically equivalent iff they are true in same models: α ≡ ß iff α ⊨ β and β ⊨ α

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

You need to know these !

Recap propositional logic: Entailment

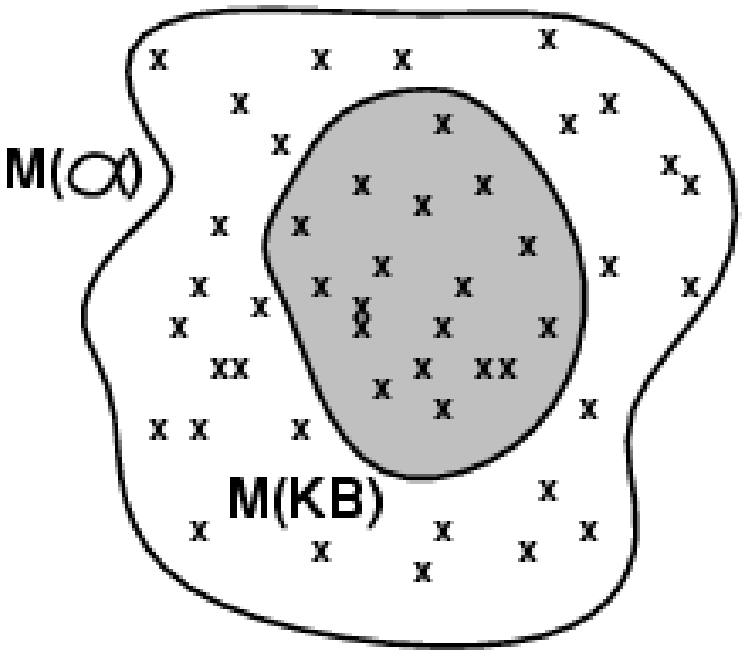
- Entailment means that one thing follows from another:

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., the KB containing "the Giants won and the Reds won" entails "The Giants won".
  - E.g., x+y = 4 entails 4 = x+y
  - E.g., "Mary is Sue's sister and Amy is Sue's daughter" entails "Mary is Amy's aunt."

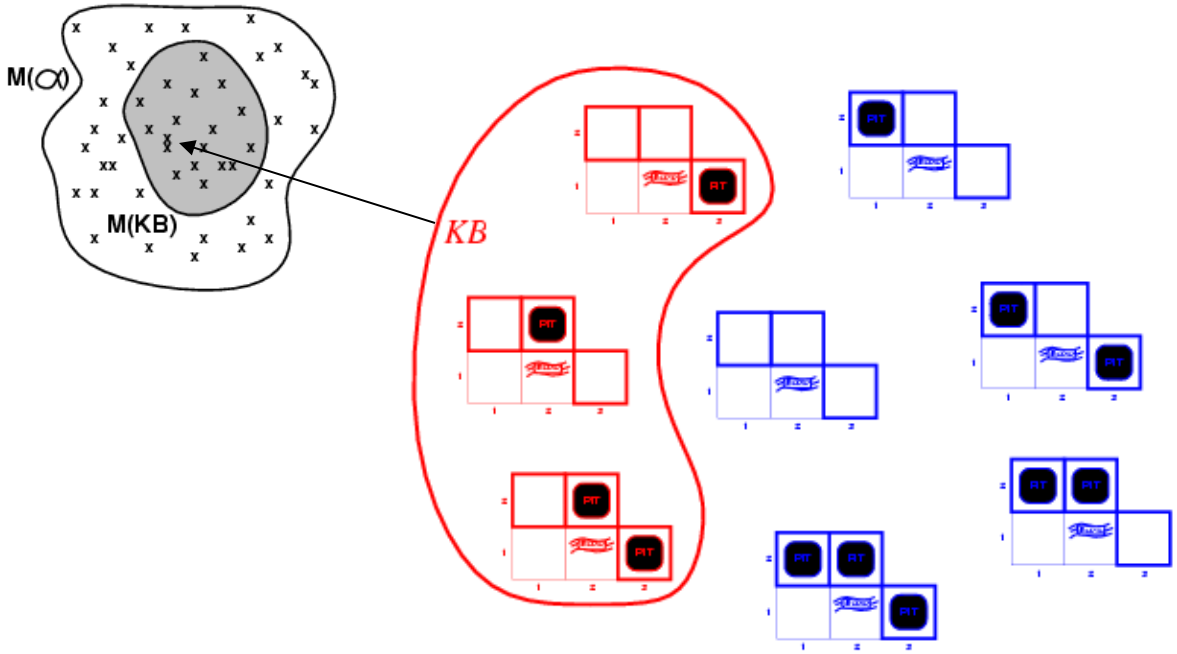
Review: Models (and in FOL, Interpretations)

- Models are formal worlds in which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then KB $\models \alpha$ iff $M(KB) \subseteq M(\alpha)$
  - E.g. *KB,* = "Mary is Sue's sister and Amy is Sue's daughter."
  - α = "Mary is Amy's aunt."



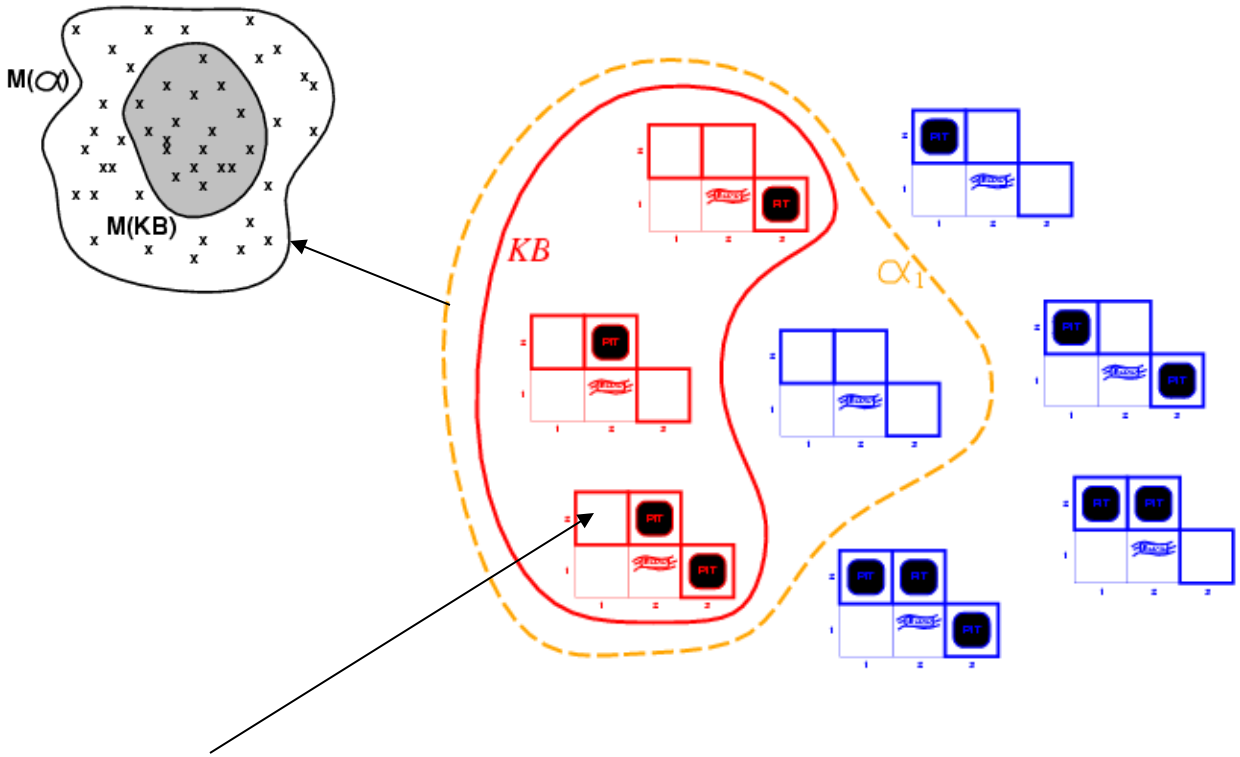
- Think of KB and α as constraints, and of models m as possible states.
- M(KB) are the solutions to KB and M(α) the solutions to α.
- Then, KB $\models \alpha$, i.e., $\models (KB \Rightarrow a)$, when all solutions to KB are also solutions to α.

Review: Wumpus models



- *KB* = all possible wumpus-worlds consistent with the observations and the "physics" of the Wumpus world.

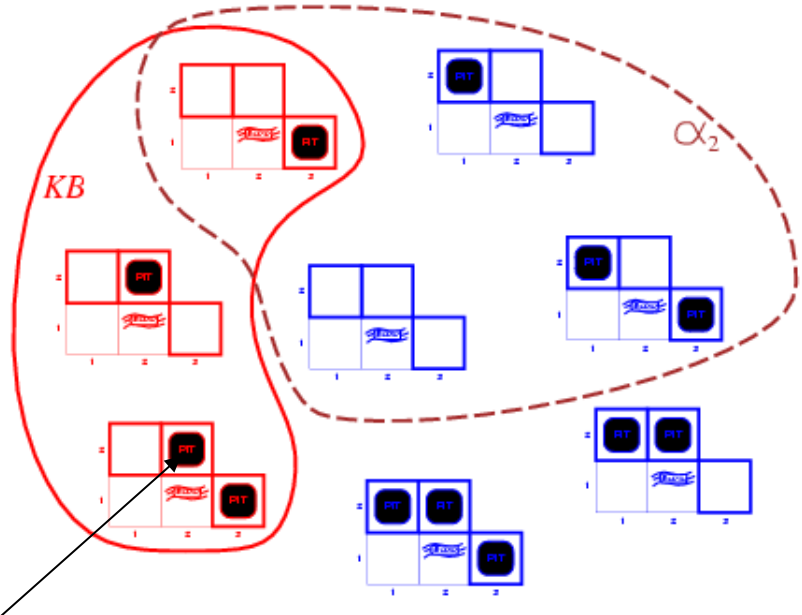
Review: Wumpus models



α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking.

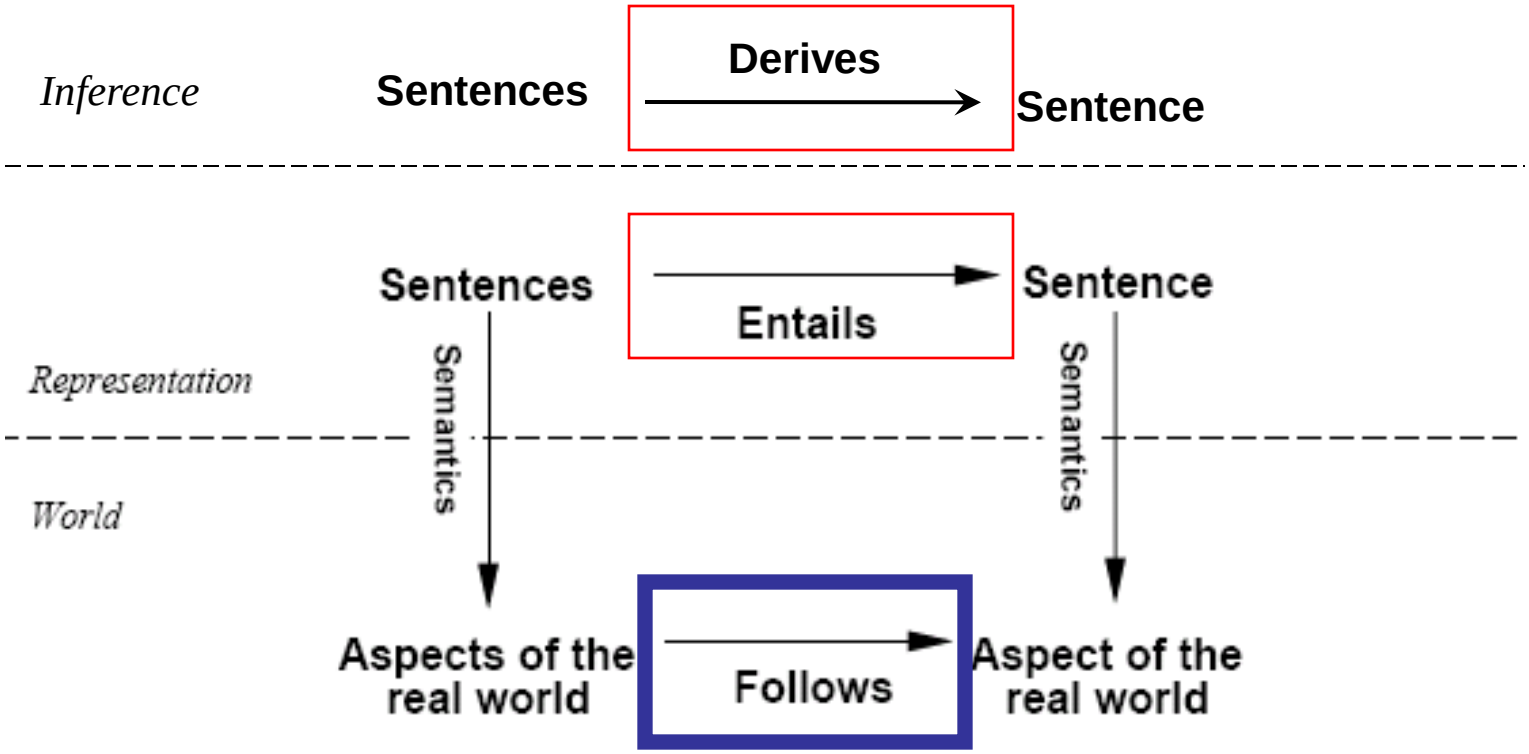
Every model that makes KB true also makes α_1 true.

Wumpus models



α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Review: Schematic for Follows, Entails, and Derives



If KB is true in the real world,

then any sentence α ***entailed*** *by KB*

and any sentence α ***derived*** *from KB*

by a sound inference procedure

is also true in the real world.

Schematic Example: Follows, Entails, and Derives

Inference

"Mary is Sue's sister and Amy is Sue's daughter."

"An aunt is a sister of a parent."

Derives →

Is it provable?

"Mary is Amy's aunt."

Representation

"Mary is Sue's sister and Amy is Sue's daughter."

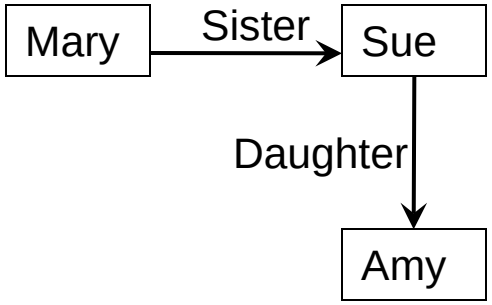
"An aunt is a sister of a parent."

Entails →

Is it true?

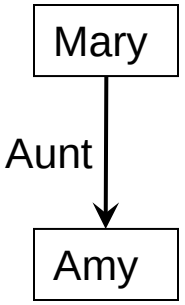
"Mary is Amy's aunt."

World



Follows →

Is it the case?



Recap propositional logic: Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is false in all models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models A$ if and only if $(KB \wedge \neg A)$ is unsatisfiable
(there is no model for which KB is true and A is false)

Inference Procedures

- *KB* $\vdash_i$ A means that sentence A can be derived from *KB* by procedure *i*

- Soundness: *i* is sound if whenever *KB* $\vdash_i \alpha$, it is also true that *KB* $\models \alpha$
  - *(no wrong inferences, but maybe not all inferences)*

- Completeness: *i* is complete if whenever *KB* $\models \alpha$, it is also true that *KB* $\vdash_i \alpha$
  - *(all inferences can be made, but maybe some wrong extra ones as well)*

- Entailment can be used for inference (Model checking)
  - enumerate all possible models and check whether α is true.
  - For *n* symbols, time complexity is $O(2^n)$...

- Inference can be done directly on the sentences
  - Forward chaining, backward chaining, resolution (see FOPC, later)

Resolution = Efficient Implication

Recall that (A => B) = ((NOT A) OR B)
and so:

(Y OR X) = ((NOT X) => Y)

((NOT Y) OR Z) = (Y => Z)

which yields:

((Y OR X) AND ((NOT Y) OR Z)) = ((NOT X) => Z) = (X OR Z)

(OR A B C D) ->Same-> (NOT (OR B C D)) => A

(OR ¬A E F G) ->Same-> A => (OR E F G)

(OR B C D E F G)

(NOT (OR B C D)) => (OR E F G)

(OR B C D E F G)

Recall: All clauses in KB are conjoined by an implicit AND (= CNF representation).

Resolution Examples

- Resolution: inference rule for CNF: sound and complete! *

$(A \vee B \vee C)$

$(\neg A)$

– – – – – – – – – – – – –

$\therefore (B \vee C)$

"If A or B or C is true, but not A, then B or C must be true."

$(A \vee B \vee C)$

$(\neg A \vee D \vee E)$

– – – – – – – – – – – –

$\therefore (B \vee C \vee D \vee E)$

"If A is false then B or C must be true, or if A is true then D or E must be true, hence since A is either true or false, B or C or D or E must be true."

$(A \vee B)$

$(\neg A \vee B)$

– – – – – – – – –

$\therefore (B \vee B) \equiv B$ ←

"If A or B is true, and not A or B is true, then B must be true."

Simplification is done always.

* Resolution is "refutation complete" in that it can prove the truth of any entailed sentence by refutation.

Only Resolve ONE Literal Pair!

If more than one pair, result always = TRUE.

Useless!! Always simplifies to TRUE!!

No!

(OR A B C D)
(OR ¬A ¬B F G)

(OR C D F G)

No! This is wrong!

No!

(OR A B C D)
(OR ¬A ¬B ¬C)

(OR D)

No! This is wrong!

Yes! (but = TRUE)

(OR A B C D)
(OR ¬A ¬B F G)

(OR B ¬B C D F G)

Yes! (but = TRUE)

Yes! (but = TRUE)

(OR A B C D)
(OR ¬A ¬B ¬C)

(OR A ¬A B ¬B D)

Yes! (but = TRUE)

Resolution Algorithm

- The resolution algorithm tries to prove: $KB \models \alpha$ *equivalent to* $KB \wedge \neg\alpha$ *unsatisfiable*

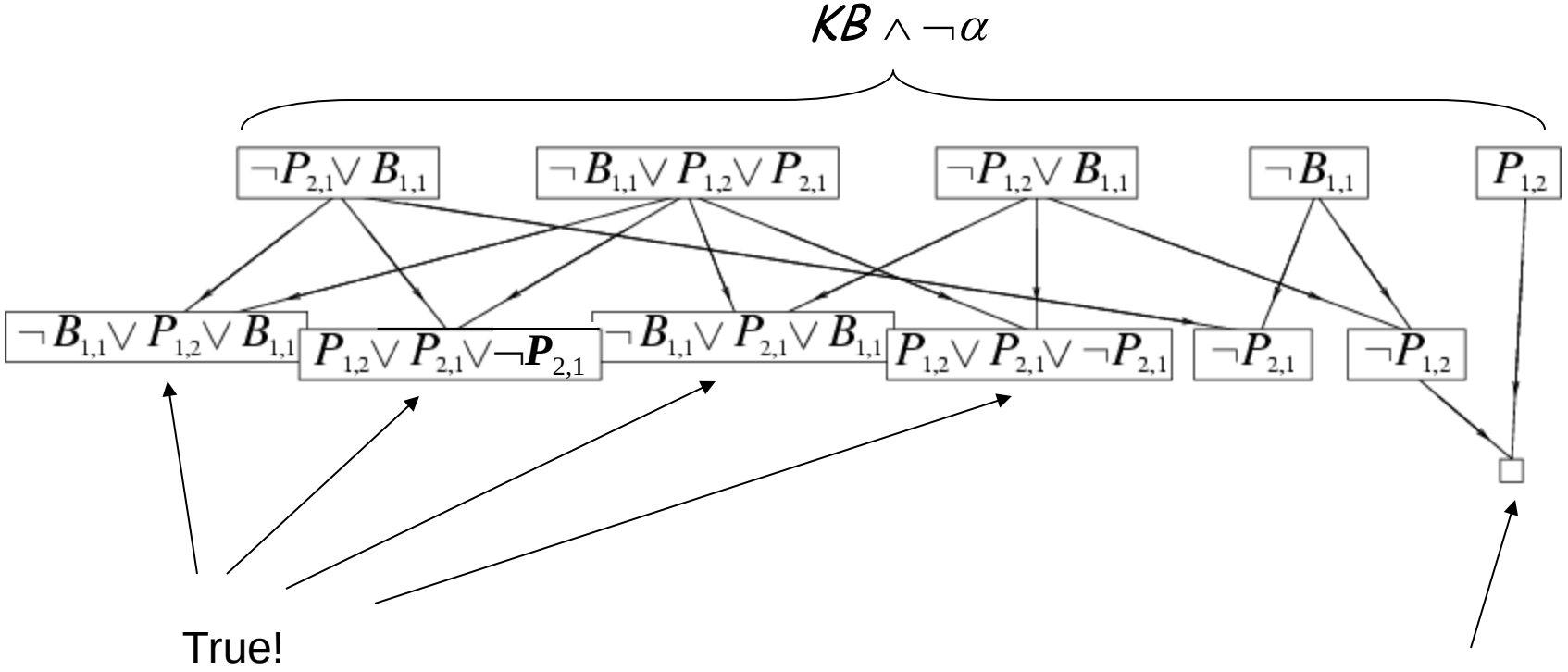
- Generate all new sentences from KB and the (negated) query.
- One of two things can happen:

1. We find $P \wedge \neg P$ which is unsatisfiable. I.e. we can entail the query.

2. We find no contradiction: there is a model that satisfies the sentence $KB \wedge \neg\alpha$ (non-trivial) and hence we cannot entail the query.

Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$
- $\alpha = \neg P_{1,2}$



Detailed Resolution Proof Example

- **In words:** *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

  Prove that the unicorn is both magical and horned.

((NOT Y) (NOT R)) **(M Y)** **(R Y)** **(H (NOT M))**
(H R) **((NOT H) G)** **((NOT G) (NOT H))**

- **Fourth, produce a resolution proof ending in ( ):**
- Resolve (¬H ¬G) and (¬H G) to give (¬H)
- Resolve (¬Y ¬R) and (Y M) to give (¬R M)
- Resolve (¬R M) and (R H) to give (M H)
- Resolve (M H) and (¬M H) to give (H)
- Resolve (¬H) and (H) to give ( )

- Of course, there are many other proofs, which are OK iff correct.

Propositional Logic --- Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions

- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
  - valid: sentence is true in every model (a tautology)

- Logical equivalences allow syntactic manipulations

- Propositional logic lacks expressive power
  - Can only state specific facts about the world.
  - Cannot express general rules about the world
    (use First Order Predicate Logic instead)

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Knowledge Representation using First-Order Logic

- Propositional Logic is **Useful** --- but has **Limited Expressive Power**
- First Order Predicate Calculus (FOPC), or First Order Logic (FOL).
  - FOPC has greatly expanded expressive power, though still limited.
- New Ontology
  - The world consists of OBJECTS (for propositional logic, the world was facts).
  - OBJECTS have PROPERTIES and engage in RELATIONS and FUNCTIONS.
- New Syntax
  - Constants, Predicates, Functions, Properties, Quantifiers.
- New Semantics
  - Meaning of new syntax.
- Knowledge engineering in FOL

Review: Syntax of FOL: Basic elements

- Constants KingJohn, 2, UCI,...
- Predicates Brother, >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
- Equality =
- Quantifiers $\forall$, $\exists$

Syntax of FOL: Basic syntax elements are symbols

- **Constant** Symbols:
  - Stand for objects in the world.
    - E.g., KingJohn, 2, UCI, ...

- **Predicate** Symbols
  - Stand for relations (maps a tuple of objects to a **truth-value**)
    - E.g., Brother(Richard, John), greater_than(3,2), ...
  - P(x, y) is usually read as "x is P of y."
    - E.g., Mother(Ann, Sue) is usually "Ann is Mother of Sue."

- **Function** Symbols
  - Stand for functions (maps a tuple of objects to an **object**)
    - E.g., Sqrt(3), LeftLegOf(John), ...

- **Model** (world) = set of domain objects, relations, functions
- **Interpretation** maps symbols onto the model (world)
  - Very many interpretations are possible for each KB and world!
  - Job of the KB is to rule out models inconsistent with our knowledge.

Syntax of FOL: Terms

- **Term** = logical expression that **refers to an object**

- **There are two kinds of terms:**

  - **Constant Symbols** stand for (or name) objects:
    - E.g., KingJohn, 2, UCI, Wumpus, ...

  - **Function Symbols** map tuples of objects to an object:
    - E.g., LeftLeg(KingJohn), Mother(Mary), Sqrt(x)
    - This is nothing but a complicated kind of name
      - No “subroutine” call, no “return value”

Syntax of FOL: Atomic Sentences

- **Atomic Sentences** state facts (logical truth values).
  - An **atomic sentence** is a Predicate symbol, optionally followed by a parenthesized list of any argument terms
  - E.g., *Married(Father(Richard), Mother(John))*
  - An **atomic sentence** asserts that some relationship (some predicate) holds among the objects that are its arguments.

- An **Atomic Sentence is true** in a given model if the relation referred to by the predicate symbol holds among the objects (terms) referred to by the arguments.

Syntax of FOL: Connectives & Complex Sentences

- **Complex Sentences** are formed in the same way, and are formed using the same logical connectives, as we already know from propositional logic

- The **Logical Connectives**:
  - $\Leftrightarrow$ biconditional
  - $\Rightarrow$ implication
  - $\wedge$ and
  - $\vee$ or
  - $\neg$ negation

- **Semantics** for these logical connectives are the same as we already know from propositional logic.

Syntax of FOL: Variables

- **Variables** range over objects in the world.
- A **variable** is like a **term** because it represents an object.
- A **variable** may be used wherever a **term** may be used.
  - **Variables** may be arguments to functions and predicates.
- A **term with NO variables** is called a **ground term**.
- All variables must be bound by a quantifier, ∀ or ∃
- (A **variable not bound by a quantifier** is called **free**.)
  - Used by mathematicians, not used in this class

Syntax of FOL: Logical Quantifiers

- There are two **Logical Quantifiers:**
  - **Universal:** $\forall$ x P(x) means "For all x, P(x)."
    - The "upside-down A" reminds you of "ALL."
  - **Existential:** $\exists$ x P(x) means "There exists x such that, P(x)."
    - The "upside-down E" reminds you of "EXISTS."

- Syntactic "sugar" --- we really only need one quantifier.
  - $\forall$ x P(x) $\equiv \neg\exists$ x $\neg$P(x)
  - $\exists$ x P(x) $\equiv \neg\forall$ x $\neg$P(x)
  - You can ALWAYS convert one quantifier to the other.

- **RULES:** $\forall \equiv \neg\exists\neg$ and $\exists \equiv \neg\forall\neg$

- **RULE:** To move negation "in" across a quantifier,
  change the quantifier to "the other quantifier"
  and negate the predicate on "the other side."
  - $\neg\forall$ x P(x) $\equiv \exists$ x $\neg$P(x)
  - $\neg\exists$ x P(x) $\equiv \forall$ x $\neg$P(x)

Universal Quantification ∀

- ∀ means “for all”
- Allows us to make statements about all objects that have certain properties
- Can now state general rules:

  ∀ x King(x) => Person(x) “All kings are persons.”

  ∀ x Person(x) => HasHead(x) “Every person has a head.”

  ∀ i Integer(i) => Integer(plus(i,1)) “If i is an integer then i+1 is an integer.”

  Note that
  ∀ x King(x) ∧ Person(x) is not correct!
  This would imply that all objects x are Kings and are People

  ∀ x King(x) => Person(x) is the correct way to say this

  Note that => is the natural connective to use with ∀ .

Existential Quantification ∃

- ∃ x means “there exists an x such that….” (at least one object x)
- Allows us to make statements about some object without naming it
- Examples:

  ∃ x King(x) “Some object is a king.”

  ∃ x Lives_in(John, Castle(x)) “John lives in somebody’s castle.”

  ∃ i Integer(i) ∧ GreaterThan(i,0) “Some integer is greater than zero.”

Note that ∧ is the natural connective to use with ∃

(And remember that => is the natural connective to use with ∀)

Combining Quantifiers --- Order (Scope)

The order of “unlike” quantifiers is important.

$\forall x \exists y$ Loves(x,y)

– For everyone (“all x”) there is someone (“exists y”) whom they love

$\exists y \forall x$ Loves(x,y)

- there is someone (“exists y”) whom everyone loves (“all x”)

Clearer with parentheses: $\exists y (\forall x$ Loves(x,y))

The order of “like” quantifiers does not matter.

$\forall x \forall y P(x, y) \equiv \forall y \forall x P(x, y)$

$\exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$

De Morgan's Law for Quantifiers

De Morgan's Rule

$$P \wedge Q \equiv \neg(\neg P \vee \neg Q)$$

$$P \vee Q \equiv \neg(\neg P \wedge \neg Q)$$

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Generalized De Morgan's Rule

$$\forall x\, P \equiv \neg\exists x\, (\neg P)$$

$$\exists x\, P \equiv \neg\forall x\, (\neg P)$$

$$\neg\forall x\, P \equiv \exists x\, (\neg P)$$

$$\neg\exists x\, P \equiv \forall x\, (\neg P)$$

Rule is simple: if you bring a negation inside a disjunction or a conjunction, always switch between them (or →and, and → or).

Fun with sentences

Brothers are siblings

$$\forall x, y \;\; Brother(x, y) \Rightarrow Sibling(x, y).$$

"Sibling" is symmetric

$$\forall x, y \;\; Sibling(x, y) \Leftrightarrow Sibling(y, x).$$

One's mother is one's female parent

$$\forall x, y \;\; Mother(x, y) \Leftrightarrow (Female(x) \wedge Parent(x, y)).$$

A first cousin is a child of a parent's sibling

$$\forall x, y \;\; FirstCousin(x, y) \Leftrightarrow \exists p, ps \;\; Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)$$

More fun with sentences

- **"All persons are mortal."**
- [Use: Person(x), Mortal (x)]

- $\forall x\ \text{Person}(x) \Rightarrow \text{Mortal}(x)$
- $\forall x\ \neg\text{Person}(x) \vee \text{Mortal}(x)$

- **Common Mistakes:**
- $\forall x\ \text{Person}(x) \wedge \text{Mortal}(x)$

- **Note that => is the natural connective to use with ∀ .**

More fun with sentences

- **“Fifi has a sister who is a cat.”**
- [Use: Sister(Fifi, x), Cat(x)]
-
- $\exists x\ \text{Sister}(\text{Fifi}, x) \wedge \text{Cat}(x)$

- **Common Mistakes:**
- $\exists x\ \text{Sister}(\text{Fifi}, x) \Rightarrow \text{Cat}(x)$

- **Note that $\wedge$ is the natural connective to use with $\exists$**

More fun with sentences

- **"For every food, there is a person who eats that food."**
- [Use: Food(x), Person(y), Eats(y, x)]

- <u>All are correct:</u>
- $\forall x \exists y \; Food(x) \Rightarrow [\, Person(y) \wedge Eats(y, x) \,]$
- $\forall x \; Food(x) \Rightarrow \exists y \, [\, Person(y) \wedge Eats(y, x) \,]$
- $\forall x \exists y \; \neg Food(x) \vee [\, Person(y) \wedge Eats(y, x) \,]$
- $\forall x \exists y \, [\, \neg Food(x) \vee Person(y) \,] \wedge [\neg \, Food(x) \vee Eats(y, x) \,]$
- $\forall x \exists y \, [\, Food(x) \Rightarrow Person(y) \,] \wedge [\, Food(x) \Rightarrow Eats(y, x) \,]$

- **Common Mistakes:**
- $\forall x \exists y \, [\, Food(x) \wedge Person(y) \,] \Rightarrow Eats(y, x)$
- $\forall x \exists y \; Food(x) \wedge Person(y) \wedge Eats(y, x)$

More fun with sentences

- **“Every person eats every food.”**
- [Use: Person (x), Food (y), Eats(x, y)]
-
- $\forall x \, \forall y \, [\text{Person}(x) \wedge \text{Food}(y)] \Rightarrow \text{Eats}(x, y)$
- $\forall x \, \forall y \, \neg\text{Person}(x) \vee \neg\text{Food}(y) \vee \text{Eats}(x, y)$
- $\forall x \, \forall y \, \text{Person}(x) \Rightarrow [\text{Food}(y) \Rightarrow \text{Eats}(x, y)]$
- $\forall x \, \forall y \, \text{Person}(x) \Rightarrow [\neg\text{Food}(y) \vee \text{Eats}(x, y)]$
- $\forall x \, \forall y \, \neg\text{Person}(x) \vee [\text{Food}(y) \Rightarrow \text{Eats}(x, y)]$
- **Common Mistakes:**
- $\forall x \, \forall y \, \text{Person}(x) \Rightarrow [\text{Food}(y) \wedge \text{Eats}(x, y)]$
- $\forall x \, \forall y \, \text{Person}(x) \wedge \text{Food}(y) \wedge \text{Eats}(x, y)$

More fun with sentences

- **"All greedy kings are evil."**
- [Use: King(x), Greedy(x), Evil(x)]
-
- $\forall x\ [\ Greedy(x) \wedge King(x)\ ] \Rightarrow Evil(x)$
- $\forall x\ \neg Greedy(x) \vee \neg King(x) \vee Evil(x)$
- $\forall x\ Greedy(x) \Rightarrow [\ King(x) \Rightarrow Evil(x)\ ]$
- **Common Mistakes:**
- $\forall x\ Greedy(x) \wedge King(x) \wedge Evil(x)$

More fun with sentences

- **“Everyone has a favorite food.”**
- [Use: Person(x), Food(y), Favorite(y, x)]
-
- $\forall x \exists y \text{ Person}(x) \Rightarrow [\text{Food}(y) \wedge \text{Favorite}(y, x)]$
- $\forall x \text{ Person}(x) \Rightarrow \exists y [\text{Food}(y) \wedge \text{Favorite}(y, x)]$
- $\forall x \exists y \neg\text{Person}(x) \vee [\text{Food}(y) \wedge \text{Favorite}(y, x)]$
- $\forall x \exists y [\neg\text{Person}(x) \vee \text{Food}(y)] \wedge [\neg\text{Person}(x) \vee \text{Favorite}(y, x)]$
- $\forall x \exists y [\text{Person}(x) \Rightarrow \text{Food}(y)] \wedge [\text{Person}(x) \Rightarrow \text{Favorite}(y, x)]$
- **Common Mistakes:**
- $\forall x \exists y [\text{Person}(x) \wedge \text{Food}(y)] \Rightarrow \text{Favorite}(y, x)$
- $\forall x \exists y \text{ Person}(x) \wedge \text{Food}(y) \wedge \text{Favorite}(y, x)$

Semantics: Interpretation

- An **interpretation** of a sentence (wff) is an assignment that maps
  - Object constant symbols to objects in the world,
  - n-ary function symbols to n-ary functions in the world,
  - n-ary relation symbols to n-ary relations in the world

- Given an interpretation, an atomic sentence has the value "true" if it denotes a relation that holds for those individuals denoted in the terms. Otherwise it has the value "false."
  - Example: Kinship world:
    - Symbols = Ann, Bill, Sue, Married, Parent, Child, Sibling, …
  - World consists of individuals in relations:
    - Married(Ann,Bill) is false, Parent(Bill,Sue) is true, …

- Your job, as a Knowledge Engineer, is to construct KB so it is true ***exactly*** for your world and intended interpretation.

Semantics: Models and Definitions

- An interpretation and possible world **satisfies** a wff (sentence) if the wff has the value “true” under that interpretation in that possible world.
- A domain and an interpretation that satisfies a wff is a **model** of that wff
- Any wff that has the value “true” in all possible worlds and under all interpretations is **valid**.
- Any wff that does not have a model under any interpretation is inconsistent or **unsatisfiable**.
- Any wff that is true in at least one possible world under at least one interpretation is **satisfiable**.
- If a wff w has a value true under all the models of a set of sentences KB then KB logically **entails** w.

Conversion to CNF

- Everyone who loves all animals is loved by someone:

$$\forall x\ [\forall y\ \mathit{Animal}(y) \Rightarrow \mathit{Loves}(x,y)] \Rightarrow [\exists y\ \mathit{Loves}(y,x)]$$

1. Eliminate biconditionals and implications

$$\forall x\ [\neg\forall y\ \neg\mathit{Animal}(y) \vee \mathit{Loves}(x,y)] \vee [\exists y\ \mathit{Loves}(y,x)]$$

2. Move $\neg$ inwards:

$$\neg\forall x\ p \equiv \exists x\ \neg p, \quad \neg\exists x\ p \equiv \forall x\ \neg p$$

$$\forall x\ [\exists y\ \neg(\neg\mathit{Animal}(y) \vee \mathit{Loves}(x,y))] \vee [\exists y\ \mathit{Loves}(y,x)]$$
$$\forall x\ [\exists y\ \neg\neg\mathit{Animal}(y) \wedge \neg\mathit{Loves}(x,y)] \vee [\exists y\ \mathit{Loves}(y,x)]$$
$$\forall x\ [\exists y\ \mathit{Animal}(y) \wedge \neg\mathit{Loves}(x,y)] \vee [\exists y\ \mathit{Loves}(y,x)]$$

Conversion to CNF contd.

3. Standardize variables: each quantifier should use a different one

$$\forall x\ [\exists y\ Animal(y) \land \neg Loves(x,y)] \lor [\exists z\ Loves(z,x)]$$

4. Skolemize: a more general form of existential instantiation.
   Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:

$$\forall x\ [Animal(F(x)) \land \neg Loves(x,F(x))] \lor Loves(G(x),x)$$

5. Drop universal quantifiers:

$$[Animal(F(x)) \land \neg Loves(x,F(x))] \lor Loves(G(x),x)$$

6. Distribute $\lor$ over $\land$:

$$[Animal(F(x)) \lor Loves(G(x),x)] \land [\neg Loves(x,F(x)) \lor Loves(G(x),x)]$$

Unification

- Recall: Subst(θ, p) = result of substituting θ into sentence p

- Unify algorithm: takes 2 sentences p and q and returns a unifier if one exists

  Unify(p,q) = θ where Subst(θ, p) = Subst(θ, q)

- Example:

  p = Knows(John,x)

  q = Knows(John, Jane)

  Unify(p,q) = {x/Jane}

Unification examples

- simple example: query = Knows(John,x), i.e., who does John know?

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}
Knows(John,x)	Knows(x,OJ)	{fail}

- Last unification fails: only because x can't take values John and OJ at the same time
  - But we know that if John knows x, and everyone (x) knows OJ, we should be able to infer that John knows OJ
- Problem is due to use of same variable x in both sentences
- Simple solution: Standardizing apart eliminates overlap of variables, e.g., Knows(z,OJ)

Unification

- To unify *Knows(John,x)* and *Knows(y,z)*,

  θ = {y/John, x/z } or θ = {y/John, x/John, z/John}

- The first unifier is more general than the second.

- There is a single most general unifier (MGU) that is unique up to renaming of variables.

  MGU = { y/John, x/z }

- General algorithm in Figure 9.1 in the text

Unification Algorithm

```
function UNIFY(x, y, θ) returns a substitution to make x and y identical
  inputs: x, a variable, constant, list, or compound expression
          y, a variable, constant, list, or compound expression
          θ, the substitution built up so far (optional, defaults to empty)

  if θ = failure then return failure
  else if x = y then return θ
  else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)
  else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)
  else if COMPOUND?(x) and COMPOUND?(y) then
      return UNIFY(x.ARGS, y.ARGS, UNIFY(x.OP, y.OP, θ))
  else if LIST?(x) and LIST?(y) then
      return UNIFY(x.REST, y.REST, UNIFY(x.FIRST, y.FIRST, θ))
  else return failure
```

```
function UNIFY-VAR(var, x, θ) returns a substitution

  if {var/val} ∈ θ then return UNIFY(val, x, θ)
  else if {x/val} ∈ θ then return UNIFY(var, val, θ)
  else if OCCUR-CHECK?(var, x) then return failure
  else return add {var/x} to θ
```

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution θ that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as $F(A, B)$, the OP field picks out the function symbol F and the ARGS field picks out the argument list (A, B).

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

1. Identify the task
   - Does the circuit actually add properly?

2. Assemble the relevant knowledge
   - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   -
   - Irrelevant: size, shape, color, cost of gates
   -

3. Decide on a vocabulary
   - Alternatives:
   -

     Type(X_1) = XOR (function)
     Type(X_1, XOR) (binary predicate)
     XOR(X_1)
     (unary predicate)

The electronic circuits domain

4. Encode general knowledge of the domain
   - $\forall t_1,t_2$ Connected$(t_1, t_2) \Rightarrow$ Signal(t_1) = Signal(t_2)
   - $\forall t$ Signal$(t) = 1 \vee$ Signal$(t) = 0$
   - $1 \neq 0$
   - $\forall t_1,t_2$ Connected$(t_1, t_2) \Rightarrow$ Connected(t_2, t_1)
   - $\forall g$ Type$(g) =$ OR $\Rightarrow$ Signal(Out(1,g)) $= 1 \Leftrightarrow \exists n$ Signal(In(n,g)) $= 1$
   - $\forall g$ Type$(g) =$ AND $\Rightarrow$ Signal(Out(1,g)) $= 0 \Leftrightarrow \exists n$ Signal(In(n,g)) $= 0$
   - $\forall g$ Type$(g) =$ XOR $\Rightarrow$ Signal(Out(1,g)) $= 1 \Leftrightarrow$ Signal(In(1,g)) $\neq$ Signal(In(2,g))
   - $\forall g$ Type$(g) =$ NOT $\Rightarrow$ Signal(Out(1,g)) $\neq$ Signal(In(1,g))

The electronic circuits domain

5. Encode the specific problem instance

$Type(X_1) = XOR$ $Type(X_2) = XOR$
$Type(A_1) = AND$ $Type(A_2) = AND$
$Type(O_1) = OR$

$Connected(Out(1,X_1),In(1,X_2))$
$Connected(Out(1,X_1),In(2,A_2))$
$Connected(Out(1,A_2),In(1,O_1))$
$Connected(Out(1,A_1),In(2,O_1))$
$Connected(Out(1,X_2),Out(1,C_1))$
$Connected(Out(1,O_1),Out(2,C_1))$
$Connected(In(1,C_1),In(1,X_1))$
$Connected(In(1,C_1),In(1,A_1))$
$Connected(In(2,C_1),In(2,X_1))$
$Connected(In(2,C_1),In(2,A_1))$
$Connected(In(3,C_1),In(2,X_2))$
$Connected(In(3,C_1),In(1,A_2))$

The electronic circuits domain

6. Pose queries to the inference procedure

   What are the possible sets of values of all the terminals for the adder circuit?

$$\exists i_1,i_2,i_3,o_1,o_2\ \text{Signal}(\text{In}(1,C_1)) = i_1 \wedge \text{Signal}(\text{In}(2,C_1)) = i_2 \wedge \text{Signal}(\text{In}(3,C_1)) = i_3 \wedge \text{Signal}(\text{Out}(1,C_1)) = o_1 \wedge \text{Signal}(\text{Out}(2,C_1)) = o_2$$

7. Debug the knowledge base

   May have omitted assertions like $1 \neq 0$

CS-271P Final Review

- **Propositional Logic**
  - (7.1-7.5)
- **First-Order Logic, Knowledge Representation**
  - (8.1-8.5, 9.1-9.2)
- **Constraint Satisfaction Problems**
  - (6.1-6.4, except 6.3)
- **Machine Learning**
  - (18.1-18.4)
- Questions on any topic
- Pre-mid-term material if time and class interest
- Please review your quizzes, mid-term, & old tests
  - At least one question from a prior quiz or test will appear on the Final Exam (and all other tests)

Review Constraint Satisfaction Chapter 6.1-6.4, except 6.3.3

- What is a CSP
- Backtracking for CSP
- Local search for CSPs

Constraint Satisfaction Problems

- What is a CSP?
  - Finite set of variables $X_1, X_2, \ldots, X_n$
  - Nonempty domain of possible values for each variable $D_1, D_2, \ldots, D_n$
  - Finite set of constraints $C_1, C_2, \ldots, C_m$
    - Each constraint C_i limits the values that variables can take,
    - e.g., $X_1 \neq X_2$
  - Each constraint C_i is a pair <scope, relation>
    - Scope = Tuple of variables that participate in the constraint.
    - Relation = List of allowed combinations of variable values.
      May be an explicit list of allowed combinations.
      May be an abstract relation allowing membership testing and listing.

- CSP benefits
  - Standard representation pattern
  - Generic goal and successor functions
  - Generic heuristics (no domain specific expertise).

CSPs --- what is a solution?

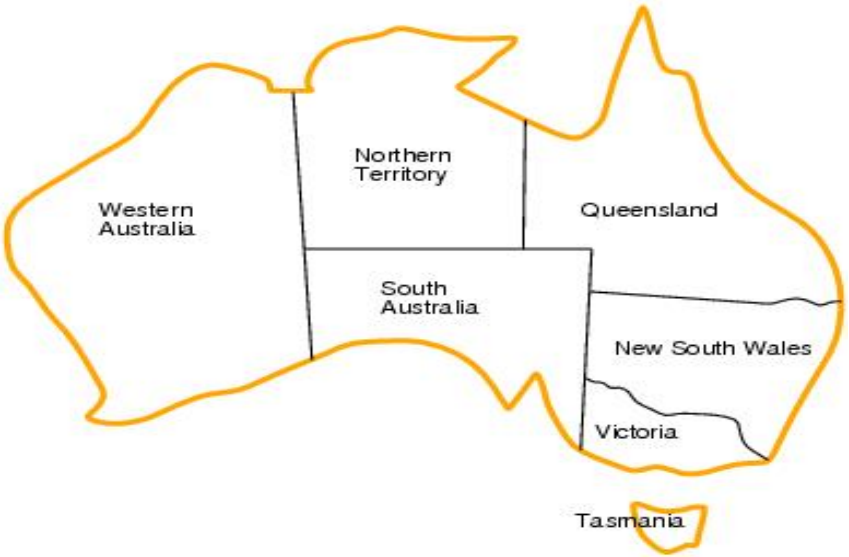
- A *state* is an *assignment* of values to some or all variables.
  - An assignment is *complete* when every variable has a value.
  - An assignment is *partial* when some variables have no values.

- ***Consistent assignment***
  - assignment does not violate the constraints

- A ***solution*** to a CSP is a complete and consistent assignment.

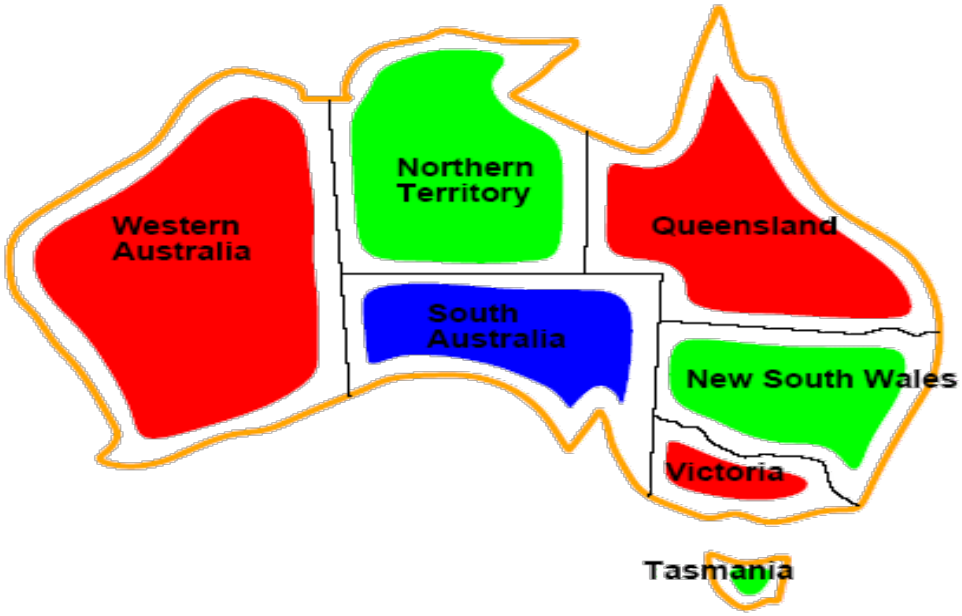
- Some CSPs require a solution that maximizes an *objective function*.

CSP example: map coloring



- Variables: *WA, NT, Q, NSW, V, SA, T*
- Domains: D_i=*{red,green,blue}*
- Constraints:adjacent regions must have different colors.
  - E.g. *WA* $\neq$ *NT*

CSP example: map coloring

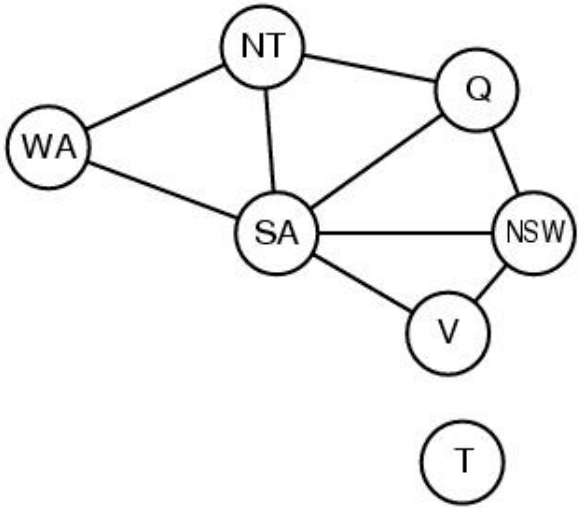


- Solutions are assignments satisfying all constraints, e.g.

  {WA=red,NT=green,Q=red,NSW=green,V=red,SA=blue,T=green}

Constraint graphs

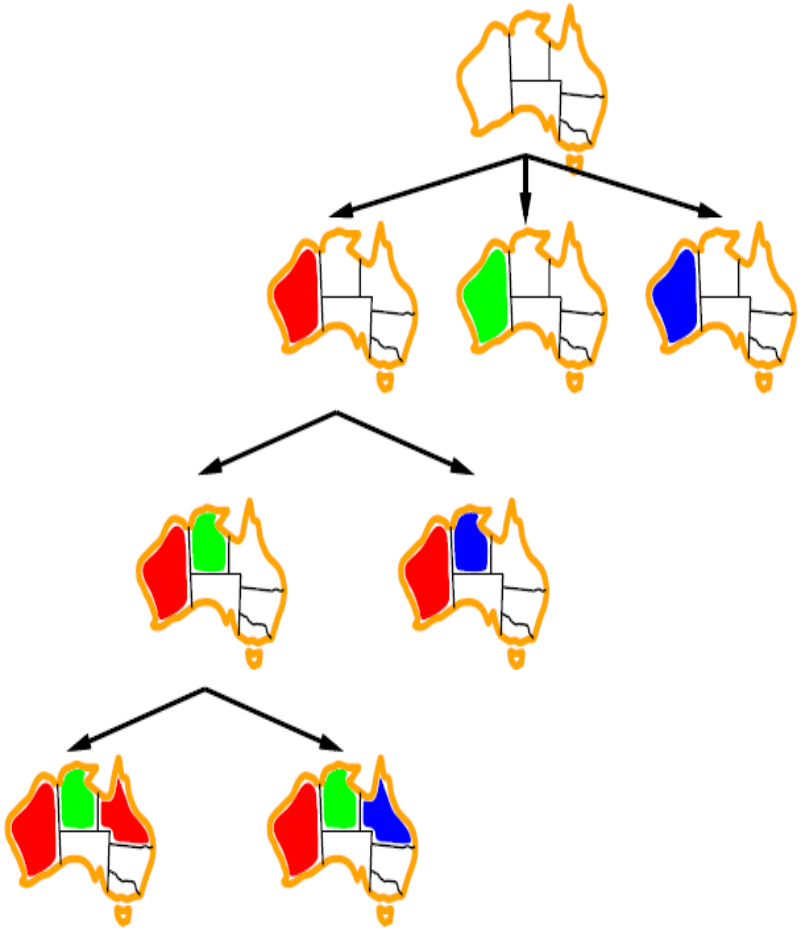
- Constraint graph:
  - nodes are variables
  - arcs are binary constraints



- Graph can be used to simplify search
  e.g. Tasmania is an independent subproblem

(will return to graph structure later)

Backtracking example

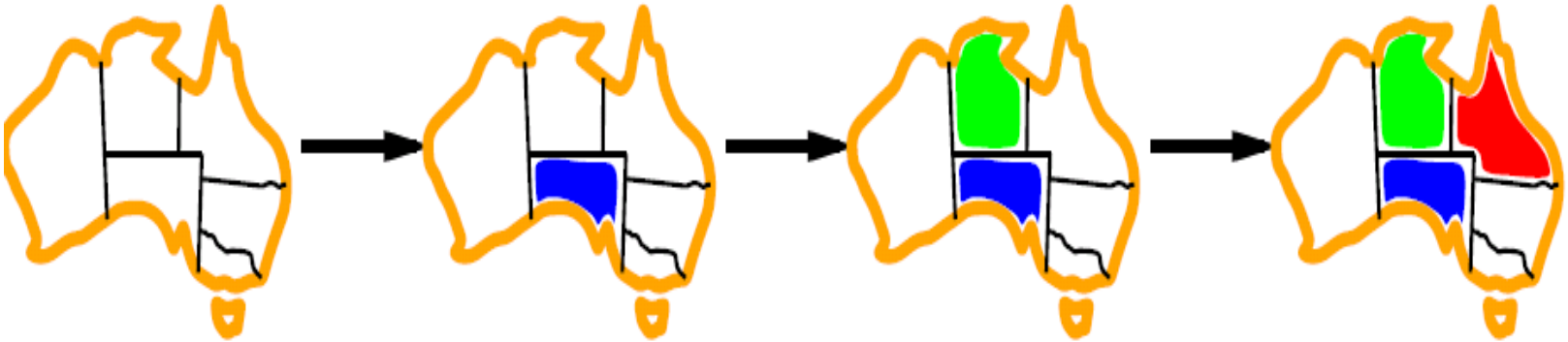


Minimum remaining values (MRV)

var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*],*assignment*,*csp*)

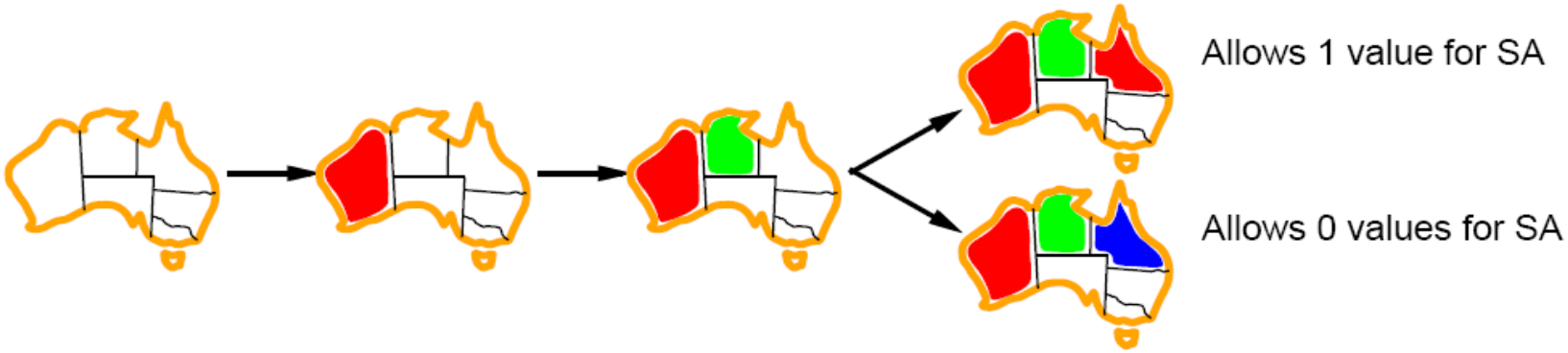
- A.k.a. most constrained variable heuristic
- *Heuristic Rule*: choose variable with the fewest legal moves
  - e.g., will immediately detect failure if X has no legal values

Degree heuristic for the initial variable



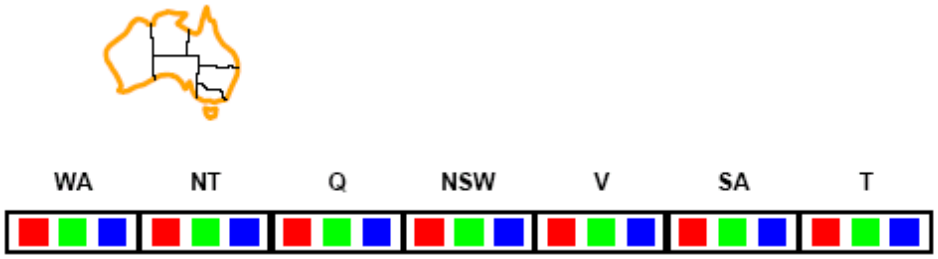
- *Heuristic Rule*: select variable that is involved in the largest number of constraints on other unassigned variables.
- Degree heuristic can be useful as a tie breaker.
- *In what order should a variable's values be tried?*

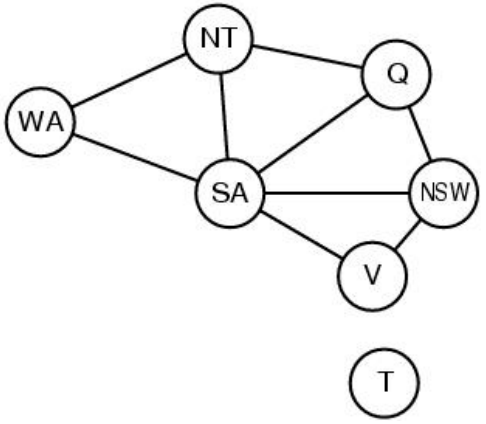
Least constraining value for value-ordering



- Least constraining value heuristic
- Heuristic Rule: given a variable choose the least constraining value
  - leaves the maximum flexibility for subsequent variable assignments

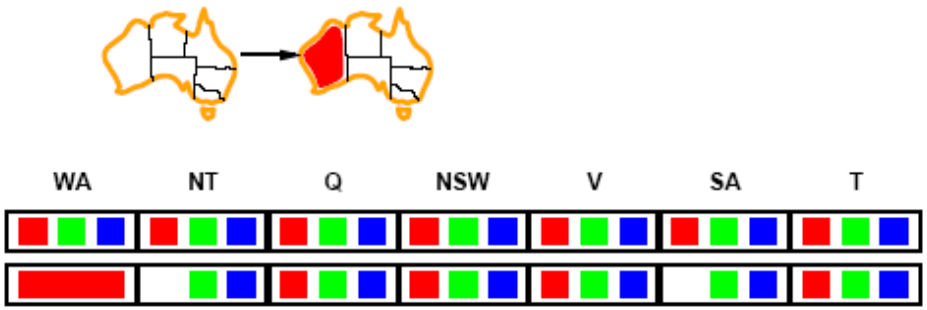
Forward checking

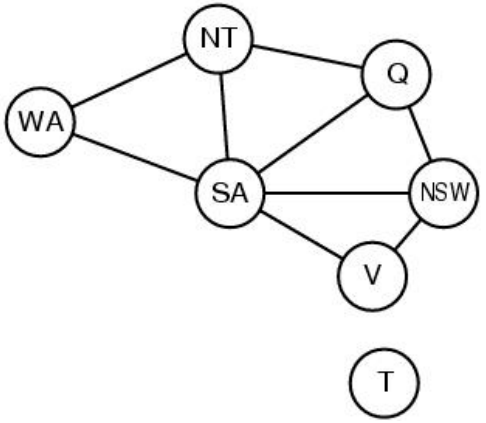




- Can we detect inevitable failure early?
  - *And avoid it later?*
- *Forward checking idea:* keep track of remaining legal values for unassigned variables.
- When a variable is assigned a value, update all neighbors in the constraint graph.
- **Forward checking stops after one step and does not go beyond immediate neighbors.**
- Terminate search when any variable has no legal values.

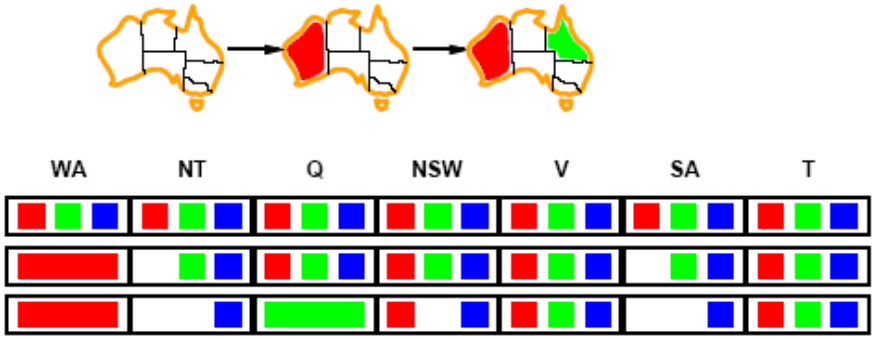
Forward checking

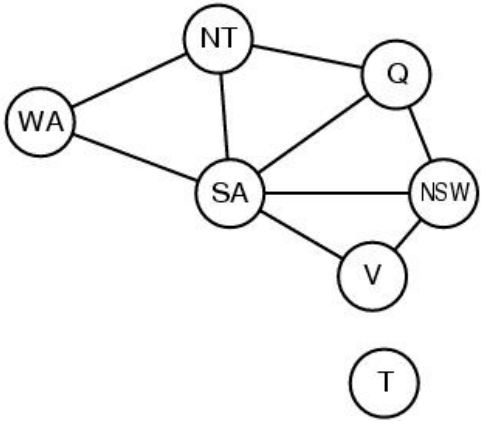




- Assign *{WA=red}*
- Effects on other variables connected by constraints to WA
  - *NT can no longer be red*
  - *SA can no longer be red*

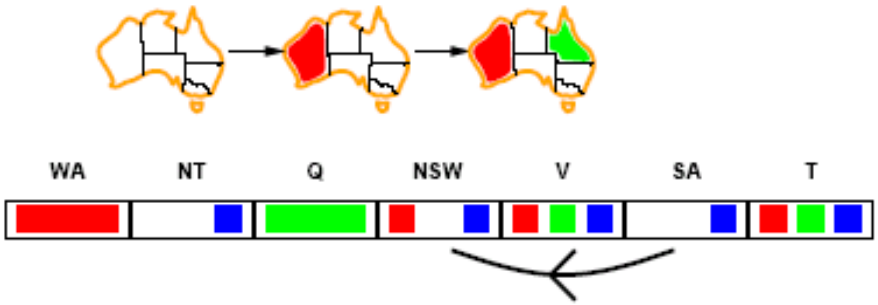
Forward checking

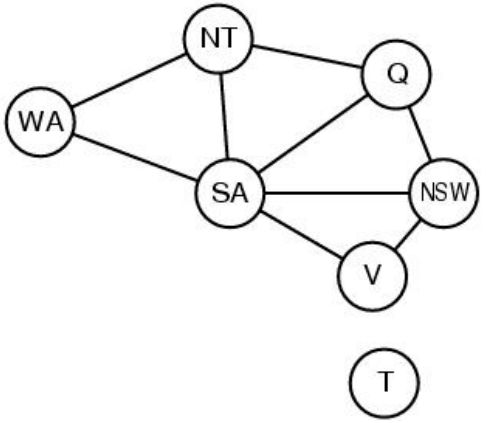




- Assign *{Q=green}*
- Effects on other variables connected by constraints with WA
  - *NT can no longer be green*
  - *NSW can no longer be green*
  - *SA can no longer be green*
- *MRV heuristic* would automatically select NT or SA next

Arc consistency





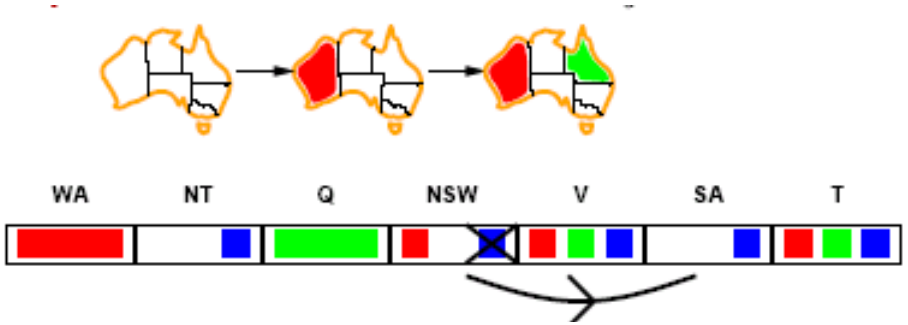
- *An Arc* $X \rightarrow Y$ is consistent if

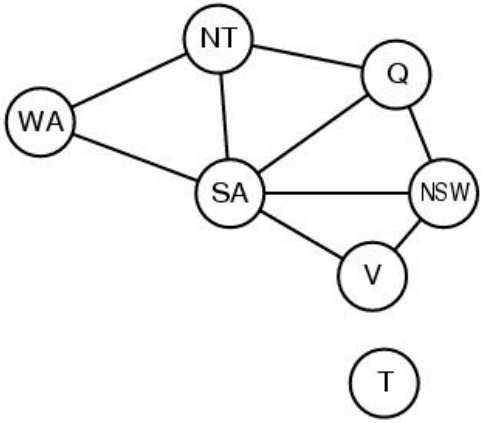
  for *every* value *x* of *X* there is some value *y* consistent with *x*

  (note that this is a directed property)

- Put all arcs $X \rightarrow Y$ onto a queue ($X \rightarrow Y$ and $Y \rightarrow X$ both go on, separately)
- Pop one arc $X \rightarrow Y$ and remove any inconsistent values from *X*
- If any change in *X,* then put all arcs $Z \rightarrow X$ back on queue, where *Z* is a neighbor of *X*
- Continue until queue is empty

Arc consistency





- $X \rightarrow Y$ is consistent if

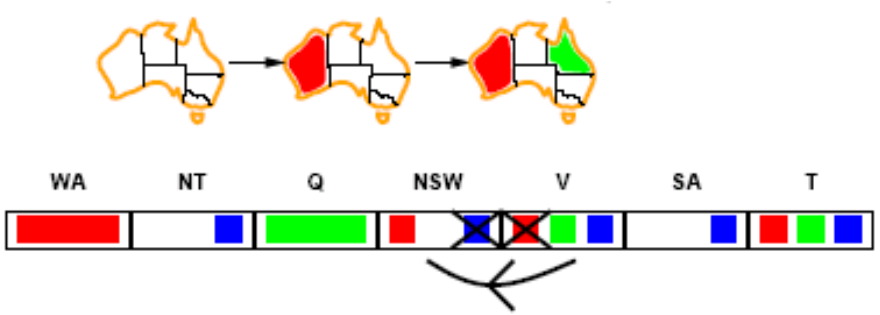
  for *every* value *x* of *X* there is some value *y* consistent with *x*

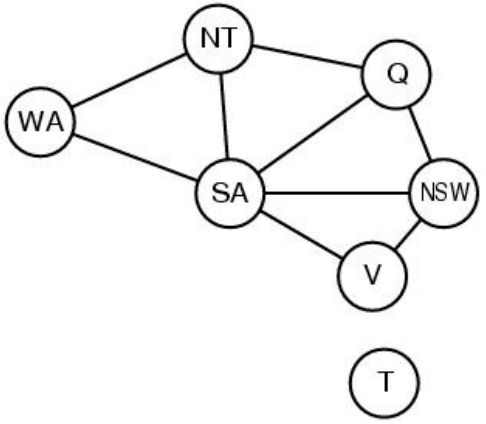
- $NSW \rightarrow SA$ is consistent if

  NSW=red and *SA=blue*

  NSW=blue and SA=???

Arc consistency



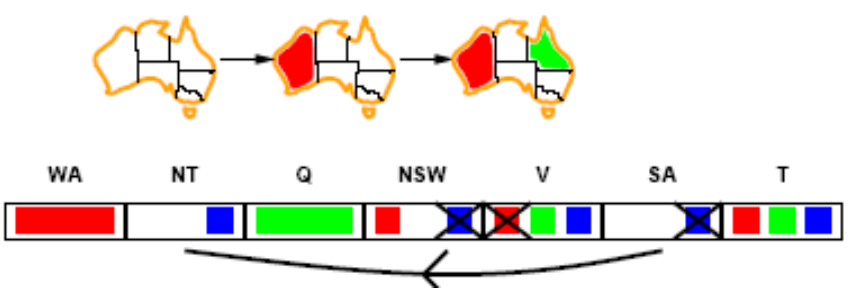


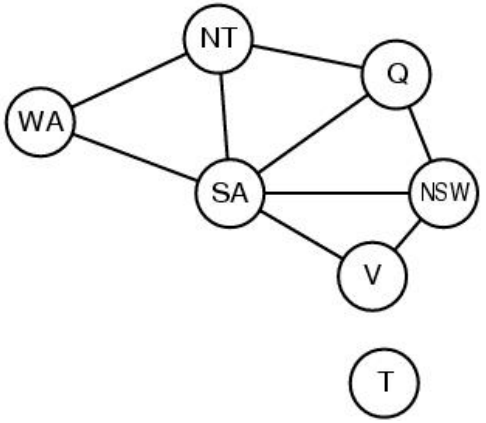
- Can enforce arc-consistency:

  Arc can be made consistent by removing *blue* from *NSW*

- Continue to propagate constraints….
  - Check $V \rightarrow NSW$
  - Not consistent for V = red
  - Remove red from *V*

Arc consistency





- Continue to propagate constraints….
- $SA \rightarrow NT$ is not consistent
  - and cannot be made consistent
- Arc consistency detects failure earlier than FC

Local search for CSPs

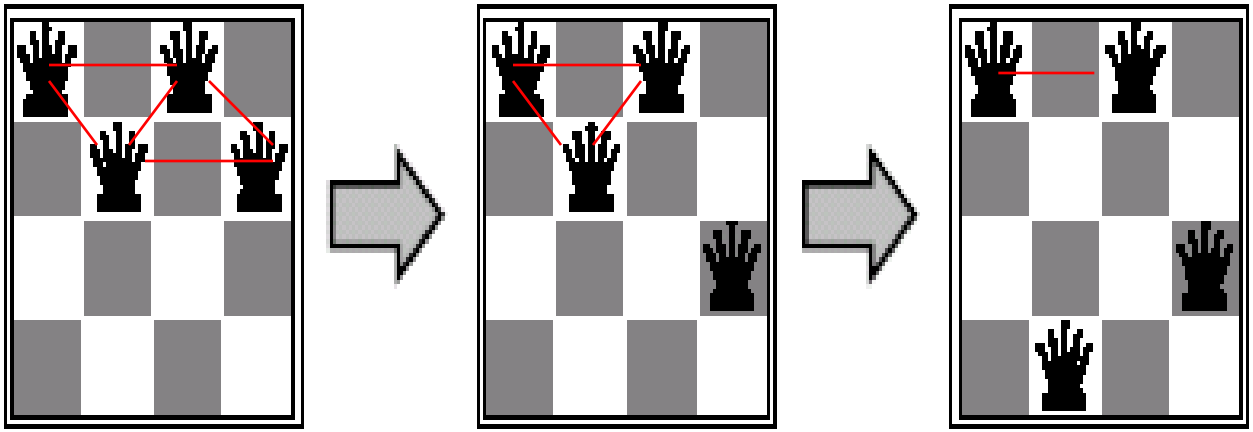
- Use complete-state representation
  - Initial state = all variables assigned values
  - Successor states = change 1 (or more) values

- For CSPs
  - allow states with unsatisfied constraints (unlike backtracking)
  - operators **reassign** variable values
  - hill-climbing with n-queens is an example

- Variable selection: randomly select any conflicted variable

- Value selection: *min-conflicts heuristic*
  - Select new value that results in a minimum number of conflicts with the other variables

Min-conflicts example 1



Use of min-conflicts heuristic in hill-climbing.

CS-271P Final Review

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The importance of a good representation

- **Properties of a good representation:**

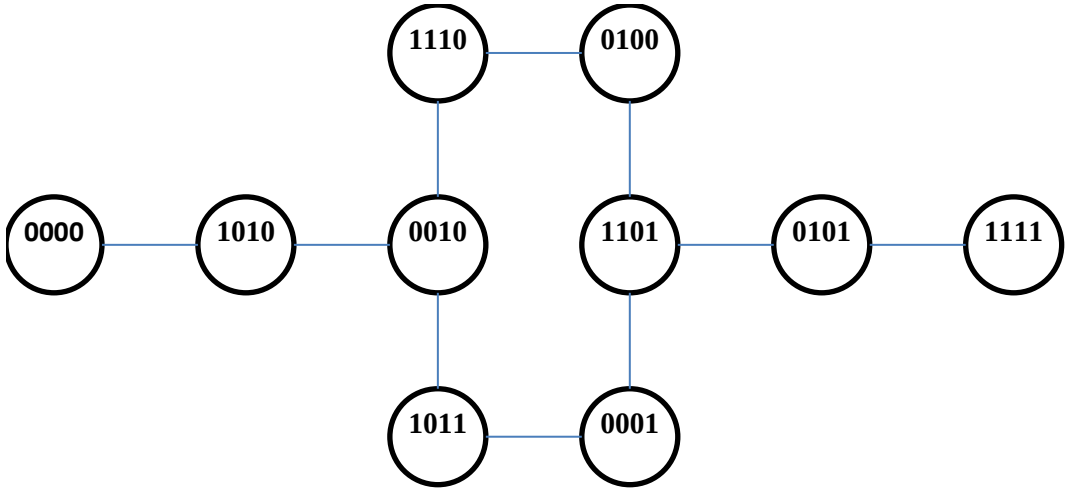
- Reveals important features
- Hides irrelevant detail
- Exposes useful constraints
- Makes frequent operations easy-to-do
- Supports local inferences from local features
  - Called the “soda straw” principle or “locality” principle
  - Inference from features “through a soda straw”
- Rapidly or efficiently computable
  - It’s nice to be fast

Reveals important features / Hides irrelevant detail

- **"You can't learn what you can't represent."** --- G. Sussman

- **In search:** *A man is traveling to market with a fox, a goose, and a bag of oats. He comes to a river. The only way across the river is a boat that can hold the man and exactly one of the fox, goose or bag of oats. The fox will eat the goose if left alone with it, and the goose will eat the oats if left alone with it.*

- **A good representation makes this problem easy:**



Terminology

- Attributes
  - Also known as features, variables, independent variables, covariates
- Target Variable
  - Also known as goal predicate, dependent variable, …
- Classification
  - Also known as discrimination, supervised classification, …
- Error function
  - Objective function, loss function, …

Inductive learning

- Let x represent the input vector of attributes
- Let f(x) represent the value of the target variable for x
  - The implicit mapping from x to f(x) is unknown to us
  - We just have training data pairs, D = {x, f(x)} available
- We want to learn a mapping from x to f, i.e.,

  h(x; θ) is "close" to f(x) for all training data points x

  θ are the parameters of our predictor h(..)
- Examples:
  - h(x; θ) = sign($w_1x_1 + w_2x_2 + w_3$)
  - h_k(x) = (x1 OR x2) AND (x3 OR NOT(x4))

Empirical Error Functions

- Empirical error function:

  $E(h) = \Sigma_x$ distance[h(x; θ) , f]

e.g., distance = squared error if h and f are real-valued (regression)
distance = delta-function if h and f are categorical (classification)

Sum is over all training pairs in the training data D

In learning, we get to choose

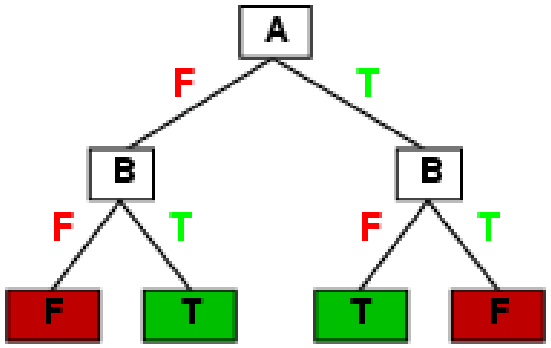
1. what class of functions h(..) that we want to learn
   – potentially a huge space! ("hypothesis space")

2. what error function/distance to use
   - should be chosen to reflect real "loss" in problem
   - but often chosen for mathematical/algorithmic convenience

Decision Tree Representations

- Decision trees are fully expressive
  - can represent any Boolean function
  - Every path in the tree could represent 1 row in the truth table
  - Yields an exponentially large tree
    - Truth table is of size 2^d, where d is the number of attributes

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



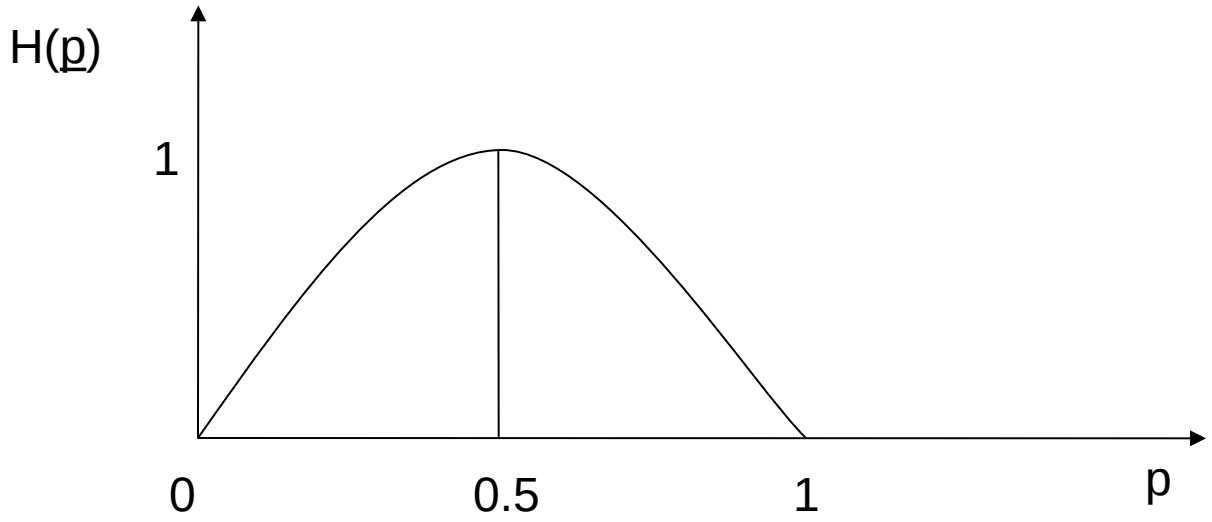
Pseudocode for Decision tree learning

```
function DTL(examples, attributes, default) returns a decision tree
    if examples is empty then return default
    else if all examples have the same classification then return the classification
    else if attributes is empty then return MODE(examples)
    else
        best ← CHOOSE-ATTRIBUTE(attributes, examples)
        tree ← a new decision tree with root test best
        for each value v_i of best do
            examples_i ← {elements of examples with best = v_i}
            subtree ← DTL(examples_i, attributes − best, MODE(examples))
            add a branch to tree with label v_i and subtree subtree
        return tree
```

Entropy with only 2 outcomes

Consider 2 class problem: p = probability of class 1, 1 – p = probability of class 2

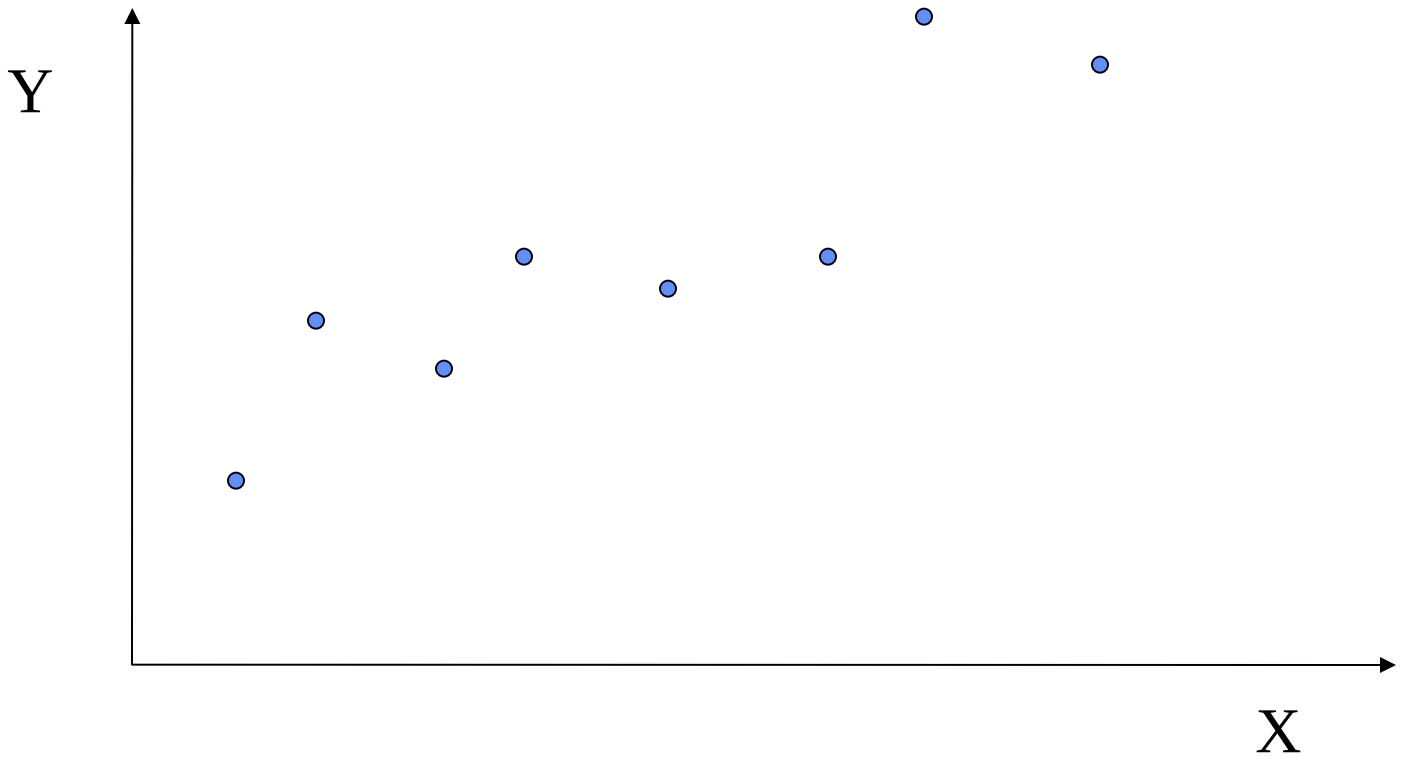
In binary case, $H(p) = - p \log p - (1-p) \log (1-p)$



Information Gain

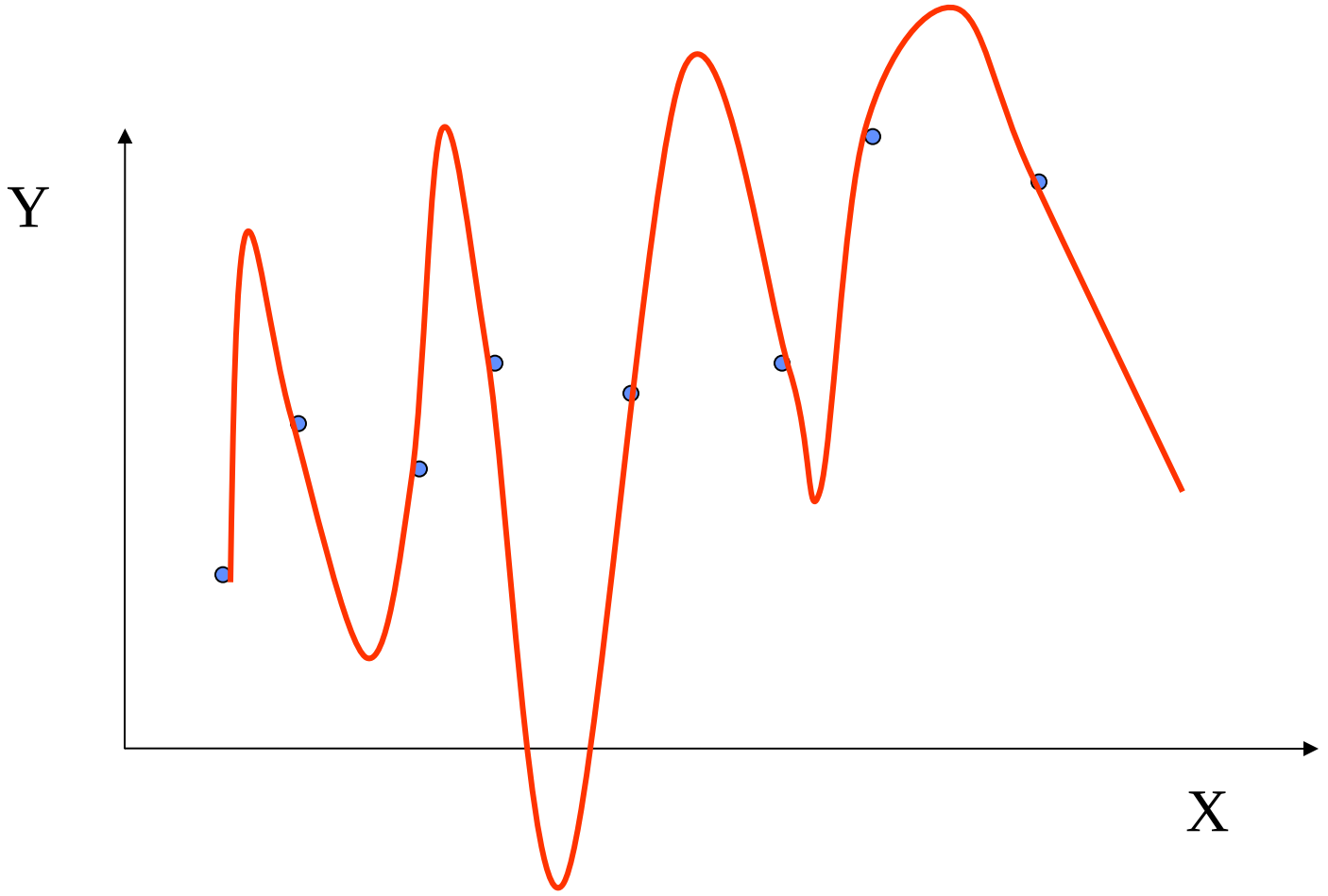
- H(p) = entropy of class distribution at a particular node
- H(p | A) = conditional entropy = average entropy of conditional class distribution, after we have partitioned the data according to the values in A
- Gain(A) = H(p) – H(p | A)
- Simple rule in decision tree learning
  - At each internal node, split on the node with the largest information gain (or equivalently, with smallest H(p|A))
- Note that by definition, conditional entropy can't be greater than the entropy

Overfitting and Underfitting

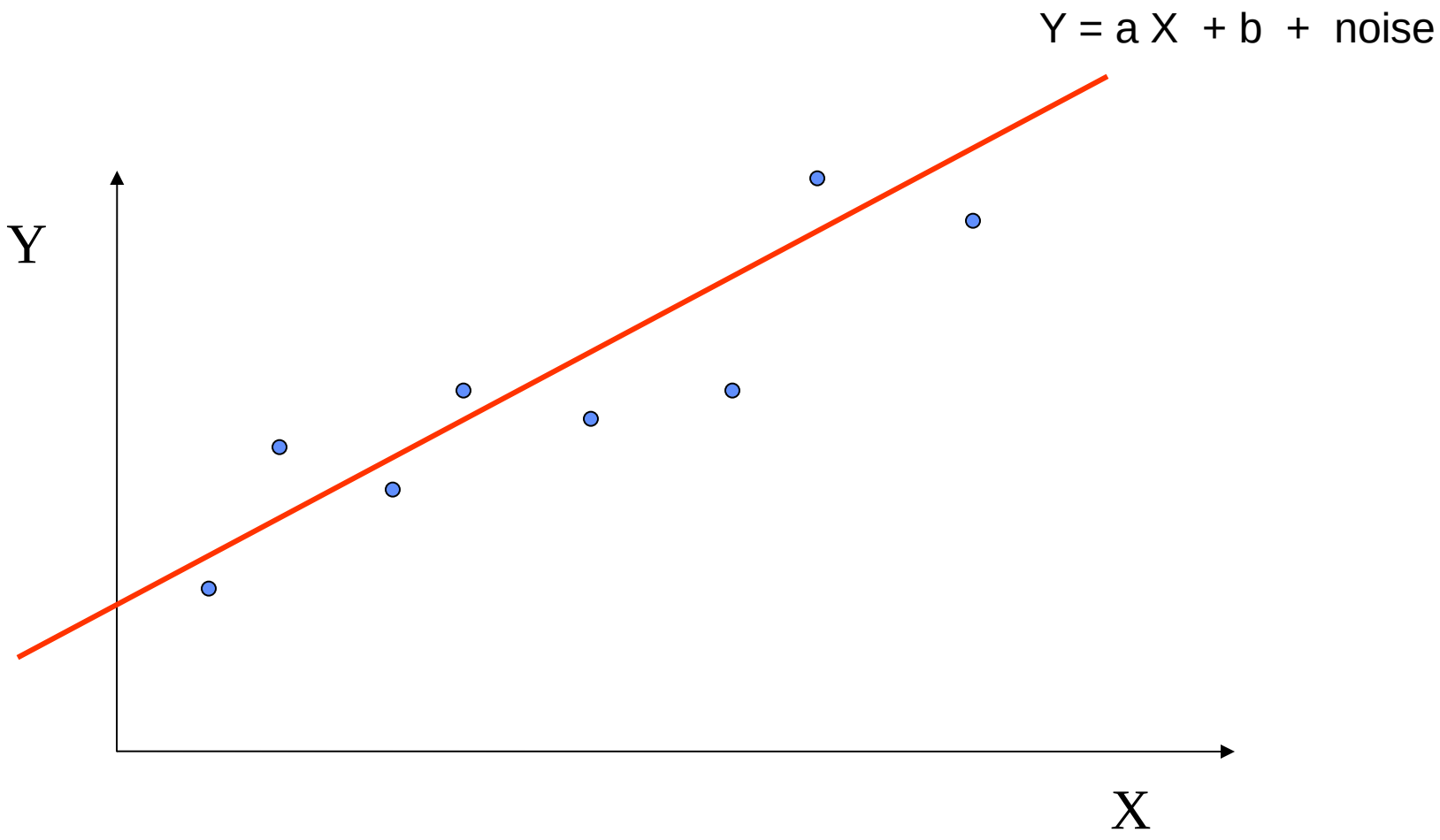


A Complex Model

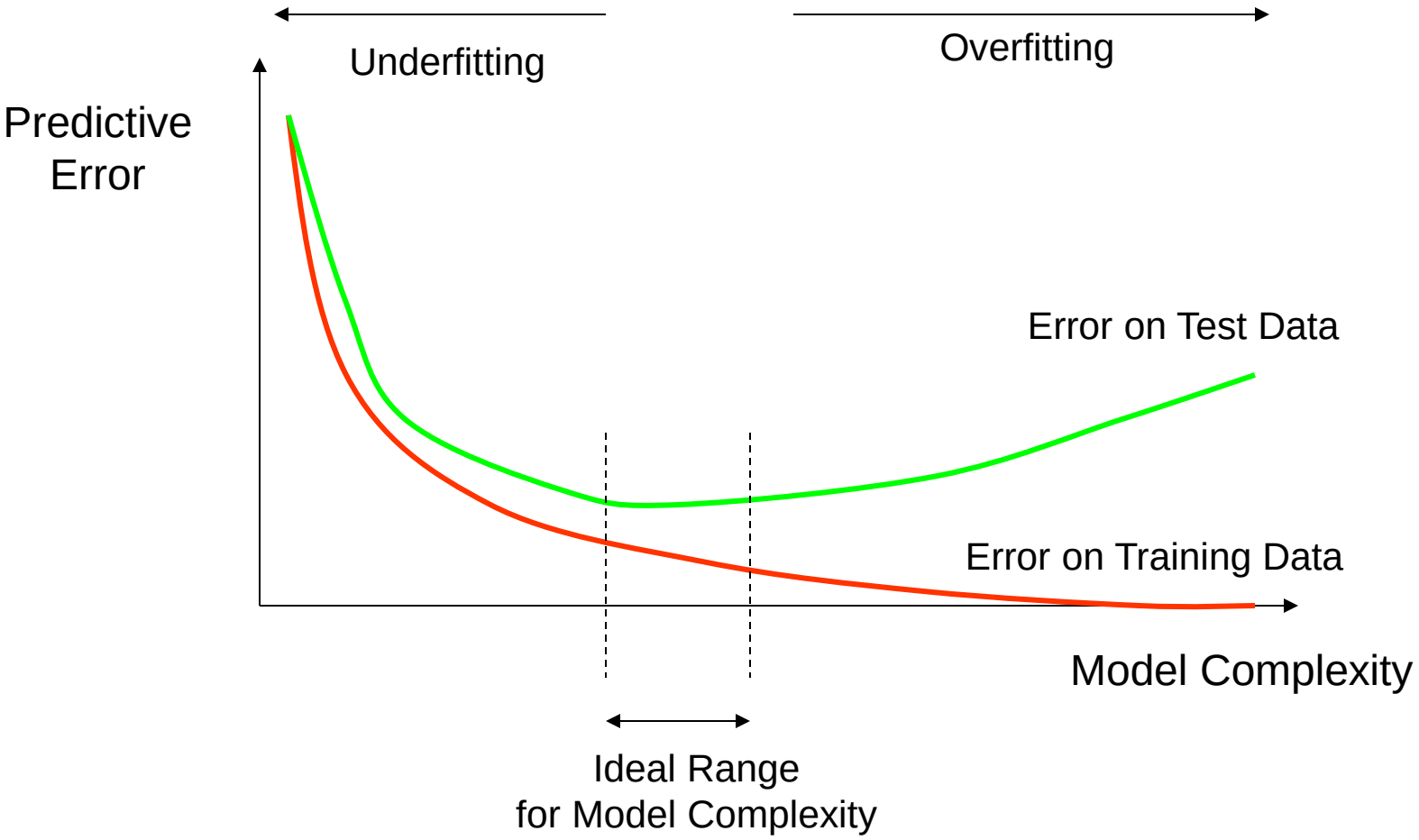
Y = high-order polynomial in X



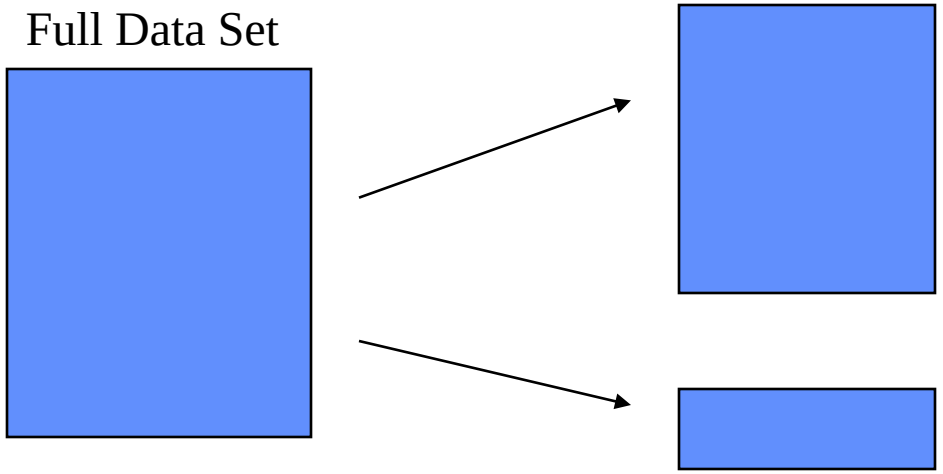
A Much Simpler Model



How Overfitting affects Prediction



Training and Validation Data



Idea: train each model on the "training data"

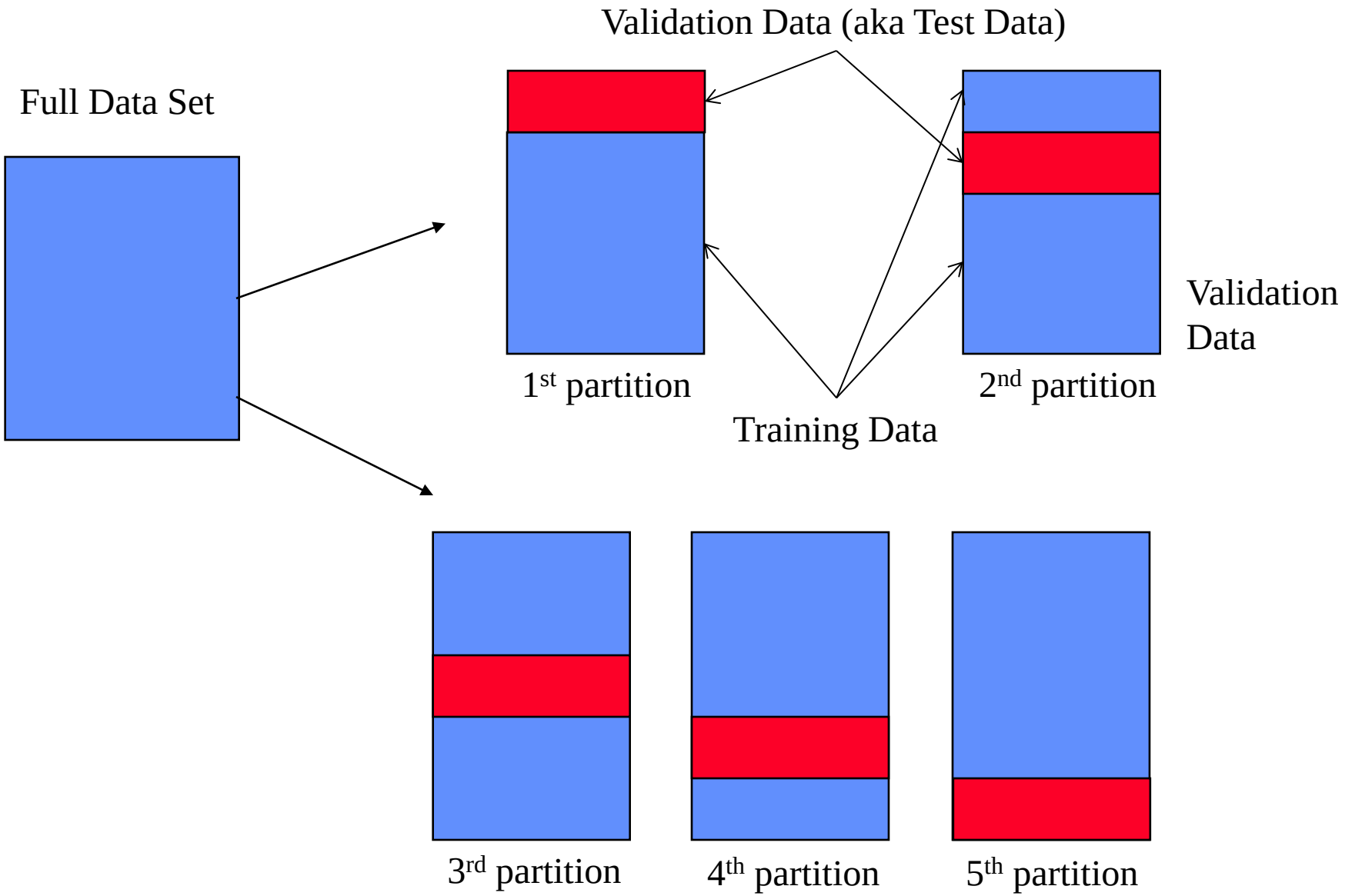
and then test each model's accuracy on the validation data

The k-fold Cross-Validation Method

- Why just choose one particular 90/10 "split" of the data?
  - In principle we could do this multiple times

- "k-fold Cross-Validation" (e.g., k=10)
  - randomly partition our full data set into k disjoint subsets (each roughly of size n/k, n = total number of training data points)
    - for i = 1:10 (here k = 10)
      - train on 90% of data,
      - Acc(i) = accuracy on other 10%
    - end
    - Cross-Validation-Accuracy = $1/k \sum_i$ Acc(i)
  - choose the method with the highest cross-validation accuracy
  - common values for k are 5 and 10
  - Can also do "leave-one-out" where k = n

Disjoint Validation Data Sets



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