Intro Linear Algebra 3A: 1st midterm

Monday October 19, 5:00- 5:50pm October 20, 2015

Short answers.

Exercise 1

(a) $[8, 4, -3, 5]^T$.

(b) $[8, 4, -3, 5]^T$ (same computation as (a)).

(c) $x_4[1, 0, -1, 1]^T + [0, 0, 1, 0]$.

(d)

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(e) Reduced row echelon form has zero row: columns do not span $\mathbf{R}^4$. Reduced row echelon form has free variables: columns not linearly independent.

Exercise 2

(a) $[1, 1, -1]^T$.

(b)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}.$$

Exercise 3 (a) False. One has $(4, 0) = T((2, 0)) \neq 2T(1, 0) = 2(1, 0)$.

(b) False, 3 vectors never span $\mathbf{R}^4$ (there will always be a zero row in the matrix with the vectors as columns.

(c) True. If $c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} = 0$ with not all $c_i = 0$, then one has $c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} + \mathbf{v_4}c_4 = 0$. The latter expression shows that $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ are still dependent.