3A: 1st midterm example solutions

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Remark: only short solutions are provided. During the exam, also give the computations. Hint: you can usually check if your answers are correct very quickly!

Exercise 1

(a) The most common answer would be $x_4[-1,0,-1,1,0]^T + x_5[-1/2,-1/2,1/2,0,1]^T + [-1/2,-1/2,3/2,0,0]^T$ (obtained from reduced row echelon form). Another possible answer is $x_4[1,0,1,-1,0]^T + x_5[-1,-1,1,0,2]^T + [0,0,1,0,-1]^T$ (this representation is not unique).

(b) From (a) it follows that the map is not one-to-one. The reduced row echelon form shows that map is **not** onto (there is a zero row).

Exercise 2

(a)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

(b) The reduced row echelon form is:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

No zero rows: onto, no free variables: one-to-one.

(c)

$$CA = \begin{bmatrix} 1 & 8 & 6 \\ -1 & 0 & -2 \end{bmatrix}$$

(d) Solve linear equations to find $S(e_1), S(e_2)$ and $S(e_3)$. The answer is:

$$B = \frac{1}{2}\begin{bmatrix} 2 & 1 & 1 \\ 4 & 1 & -1 \end{bmatrix}$$

Exercise 3

(a) False. If $m > n$, then m vectors in $\mathbf{R}^n$ are always linearly dependent (theorem in book).

(b) False. The map with the matrix $[0,0,0,0,0]$ is not surjective.

(c) False. If linear, then $T(0) = 0$, but $T(0) = [0,0,1]$.

(d) False: If linear, then $T([1,1,0]^T) = T([1,0,0]) + T([0,1,0])$, and this property does not hold.