3A: Extra exercises 3

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 (3 points)
Show that there is a unique linear map $T : \mathbf{R}^3 \to \mathbf{R}^3$ with

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

and compute the standard matrix A corresponding to this map.

Exercise 2 (7 points)
Let $c \in \mathbf{R}$ and let

$$A_c = \begin{bmatrix} 2 & 0 & 10 \\ 0 & 8+c & -3 \\ 0 & 4 & c+1 \end{bmatrix}.$$

(a) Find all values of c such that A_c is invertible. Make sure that you completely justify your answer (2 points).
(b) Compute the inverse of A_c when $c = -3$ (2 points).
(c) For all c such that A_c is not invertible, find all solutions of the equation $A_c\mathbf{x} = \mathbf{0}$. (2 points).
(d) Compute A_c^2 when $c = 0$. (1 point).