
Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 (3 points)
Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 3 & 1 & 2 \\ -1 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Compute the determinant of A using row reductions. (1.5 points)
(b) Compute the determinant of A using cofactor expansions. (1.5 points)

Exercise 2 (4 points)
Let

$$A = \begin{bmatrix} 2 & 1 & 3 & 3 \\ 0 & 1 & 3 & 2 \\ 1 & 2 & 4 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}.$$

(a) Find the determinant of A (1 points).
(b) Compute the solution to $A\mathbf{x} = [1, 0, 0, 1]^T$ using Cramer's rule (1.5 points).
(c) Compute the second column of the inverse of A using determinants (Theorem 8 in Section 3.3, 1.5 points).

Exercise 3 (3 points)
Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}.$$

(a) Compute the characteristic polynomial of A (1 point).
(b) Compute the eigenvalues of A (1 point).
(c) Use the results from (a) to answer the following question: is A invertible? (1 point)