

3A: Extra exercises 7

Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 ($10 = 2.5 + 1 + 0.5 + 1.5 + 1 + 1.5 + 1 + 1$ points)

Let

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -1 \end{bmatrix}, \mathbf{u}_4 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \end{bmatrix}.$$

Let $H = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$. Finally, let $\mathbf{u} = [1, 0, 0, 0]^T$.

- (a) Find an orthogonal basis for H .
- (b) Find an orthonormal basis for H .
- (c) What is the dimension of H ? (hint: it is not 4)
- (d) Find the orthogonal projection \mathbf{u} on H .
- (e) Find the distance between \mathbf{u} and H .
- (f) Find an orthonormal basis for H^\perp .
- (g) What is the distance between \mathbf{u} and H^\perp ?
- (h) (challenging) Explain how you could have deduced the answer to (f) from the answer in (d), and the answer to (g) from the answer to (e).