**Intro Linear Algebra 3A: midterm 2**
Friday November 13, 5:00- 5:50pm November 13, 2015

---

Short answers.

**Exercise 1**
(a) $4c - 4$. Matrix is invertible if $c \neq 1$.
(b)
$$\begin{bmatrix} -1 & -\frac{1}{2} & 1 \\ 1 & 1 & -1 \\ \frac{3}{2} & \frac{3}{4} & -1 \end{bmatrix}.$$
(c) $[1, -3, -1]^T$ (just multiply $A_0^{-1}$ by $b$).

**Exercise 2**
(a) $[1, -2, 1, 0, 0]^T, [2, -3, 0, 1, 0]^T, [3, -4, 0, 0, 1]^T$. Dimension is 3.
(b) Pivot columns of matrix: $[1, 6, 11]^T, [2, 7, 12]^T$, dimension 2, which is by definition the same as the rank.
(c) Free variables: not one-to-one, zero row: not onto.
(d) Relations preserved in reduced row echelon form: $-3[1, 6, 11]^T + 4[2, 7, 12]^T = [5, 10, 15]^T$.

**Exercise 3**
(a) Characteristic polynomial is $(2 - \lambda)(2 - \lambda)(1 - \lambda)$ (easy to compute). Hence eigenvalues are 2 and 1.
(b) For 2: $\text{Span}([1, 0, 0]^T, [0, 1, 3]^T)$. For 1: $\text{Span}([1, 0, -1/2]^T)$.
(c) Yes (answer not unique):
$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 3 & -1/2 \end{bmatrix}$$

**Exercise 4**
(a) False, if $b \neq 0$, then 0 is not in the solution set.
(b) True. $B$ is invertible if and only if $B^T$ is invertible. $B^T$ is invetible, if and only if its columns span $\mathbf{R}^n$ (invertible matrix theorem).
(c) False: If $n \neq m$, this statement is false (statement is true for $= m$, see book).
(d) True: $\det(AB) = \det(A)\det(B) = \det(B)\det(A) = \det(BA)$.
(e) True: invertible if and only if nullspace is 0 if and only if 0 is not an eigenvalue.
(f) True: characteristic polynomial is of degree $n$, has at most $n$ different zeros.