

### 3A: Extra exercises 1

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Remark: the exercise below will be graded carefully. Give explanations! and computations.

**Exercise 1** Consider the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 0$$

$$x_1 - x_2 - x_3 + x_4 = 4$$

$$x_1 + 3x_2 - 2x_3 + 2x_4 = 9$$

$$x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 = 1$$

- (a) Construct the augmented matrix  $B = [A \ \mathbf{b}]$  of the system above (here  $\mathbf{b}$  is just a vector, and  $A$  is a  $4 \times 5$  matrix).
- (b) Compute the reduced row echelon form of  $B$ .
- (c) What are the pivots and pivot columns of the matrix  $B$ ? Which variables are free?
- (d) Find all solutions to the equation (express the solutions in terms of the free variables).
- (e) If there are infinitely many solutions, give 2 different solutions.
- (f) Which vectors  $\mathbf{b}' \in \mathbf{R}^4$  are in the span of the columns of  $A$ ?
- (g) For which vectors  $\mathbf{b}' \in \mathbf{R}^4$  is the system  $[A \ \mathbf{b}']$  inconsistent?