Remark: the exercise below will be graded carefully. Give explanations and computations.

Exercise 1 (4 points)
(a) Let A be an $n \times n$ matrix which in invertible. Prove that $A^T A$ is invertible. Be sure to justify each step in your proof completely.
(b) Find a 2×2 matrix A with $\text{Nul}(A) = \text{Col}(A)$. Does such an example exist when A is a 3×3 matrix?

Exercise 2 (6 points)
Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Compute the reduced row echelon form of A. (2 points)
(b) Find a basis of the null space of A. (1 point)
(c) What is the dimension of the null space of A? (1/2 point)
(d) Find a basis of the column space of A. (1 point)
(e) What is the rank of A? (1/2 point)
(f) Find all possible subsets of the columns of A which form a basis of the column space of A (tricky, 1 point). For example, columns 4, 5, 6 also form a basis for the column space of the matrix (and there are many more).